St.Philomena's College (Autonomus), Mysore

PG Department of Mathematics

Question Bank (Revised Curriculum 2018 onwards)

First Year - Second Semester (2018 -20 Batch)

Course Title (Paper Title): Algebra-II Q.P.Code-57101

Unit	S.No	Question	Marks
1	1	Show that cancellation law holds good in an integral domain.	2m
1	2	Show that every finite integral domain is a field.	2m
1	3	Give an example of a skew field which is not a field	2m
1	4	If R is a Principal ideal domain,Does Every Prime ideal is maximal	2m
1	5	Is every subring is an ideal ? Justify	2m
1	6	If R is a ring with identity and I is an ideal of R that contains a unit then show that I=R	2m
1	7	Show that if the ring R is either real or rational then the only isomorphism from ring R to R is only identity map	2m
1	8	Define an embedding with an example	2m
1	9	Define field of quotients of the ring R and hence find the field of quotients of Gaussian integers	2m
2	10	As a consequence of Fundamental theorem of Homomorphism show that R/R=0 and R/0=R	2m

2	11	Is every prime ideal is maximal? Justify	2m
2	12	Define the degree of a polynomial.Show that $deg[f(x)g(x)] \leq degf(x) + deg(x), wheref(x)g(x)$ is non zero polynomial. When does the equality holds justify?	2m
2	13	If R is an integraldomain show that R[x] is an integral domain	2m
2	14	If R is a commutative ring with unity ,then show that $a_0 + a_1x + a_2x^2 + ... + a_nx^n$ is a nilpotent in R[x] if $a_o, a_1, , , a_n$ are nilpotent elements of R	2m
2	15	If R is a commutative ring with identity,then show that $a_0 + a_1x + a_2x^2 + ... + a_nx^n$ is a unit if a_0 is a unit and $a_1, a_2...a_n$ are nilpotent elements of R	2m
2	16	If I is an maximal ideal of R ,does I[x] is maximal ideal of R[x].Justify?	2m
2	17	Show that if $a\ and\ b$ are associates if and only if they differ by a unit	2m
2	18	Show that $(2, x)$ is maximal ideal of $Z[x]$	2m
2	19	Is $\mathbb{Z}[x]$ is a principal ideal domain ? justify	2m
2	20	If $\mathbb{F}$ is a field show that $\mathbb{F}[x]$ is an eucliden domain	2m
2	21	Show that $7 + 2i\ and\ 3 - 4i$ are coprimes in $\mathbb{Z}[i]$	2m
2	22	In a PID ,prove that every irreduicable element is a prime	2m
2	23	Justify the statement " An UFD is a PID "	2m
2	24	Define monic and primitive polynomial with an example	2m
2	25	Show that $x^n - p\ \ is\ irreducible\ in\ \mathbb{Q}[x]$	2m

2	26	State factor theorem	2m
2	27	Define Characterstic of a field $\mathbb{F}$	2m
2	28	Define Cyclotomic polynomial.Show that it is irreducible in $\mathbb{Z}[x]$	2m
3	29	Define field extension with an example	2m
3	30	Define prime subfield of a field $\mathbb{F}$ and hence find the prime subfield of $\mathbb{Q}$	2m
3	31	Define Finite and infinite extension with an example	2m
3	32	Define the degree of the field extension and hence find the degree of $[\mathbb{Q}(\sqrt{2}) : \mathbb{Q}]$	2m
3	33	Define algebraic extension with an example	2m
3	34	Define minimal polynomial with an example	2m
4	35	Define splitting field with an example Define normal extension with an example	2m
4	37	Define quadratic extension with an example	2m
4	38	Show that every quadratic extension is a algebraic extension	2m
4	39	Define Perfect field	2m
1	40	Define Separable and inseparable extension with an example	2m
4	41	Does there exists a field of order 1000? Justify.	2m
2	42	Show that every euclidean domain is a principal ideal domain	4m
4	43	Prove that $\mathbb{Q}(\sqrt{2}, \sqrt{3}) = \mathbb{Q}(\sqrt{2} + \sqrt{3})$	4m

4	44	Prove that the multiplicative group of a non zero elements of a finite field is cyclic .	4m
1	45	If R is a commutative ring with unity ,Show that R is a field if and only if the only ideals of R are (0) *and R*	6m
1	46	If $a^3 = a,\ \forall a \in R$ Show that R is a commutative	6m
3	47	If F is a finite field show that $\|F\|$ is a power of a prime number.	6m
4	48	Explain the construction of field with four elements	6m
1	49	If R is a commutative ring with identity then show that the ideal M is a maximal ideal if and only if R/M is a field.	7m
1	50	If R is a commutative ring with identity then show that the ideal P is a prime ideal if and only if R/P is an integral domain	7m
1	51	State and Prove First isomorphism theorem	7m
1	52	State and Prove Second isomorphism theorem	7m
2	53	If R is acommutative ring with identity show that $p$is a prime element if and only if (p) is a prime ideal.	7m
2	54	In a Principal ideal domain show that gcd of any two elements always exists	7m
2	55	*State and prove Gauss Lemma*	7m
2	56	If f is primitive and g is primitive show that their product is also a primitive poynomial.	7m

2	57	Define a nilradical $N(R)$ of a commutative ring R. Show that $N(R)$ is an ideal of R.	7m
3	58	If R is a commutative ring with identity show that 'x' is irreducible if and only if (x) is a maximal ideal of R	7m
3	59	State and prove Eisentien's criterion	7m
3	60	Describe an example of infinite algebraic extension	7m
3	61	State and prove rational root theorem	7m
3	62	Show that $ch\mathbb{F} = 0\ or\ p;\ where\ p\ is\ a\ prime\ number$	7m
3	63	Prove that prime subfield K is isomorphic to $\mathbb{Q}\ or\ \mathbb{Z}_p$accordingly $ChK = 0\ or\ ChK = P$	7m
3	64	Prove that every finite extension is an algebraic extension.How about the converse?Justify.	7m
4	65	If $f(x) \in F[x]$ is a polynomial of degree n then prove that therte is an extension K of F which is a splitting field of $f(x)$ such that $[K : F] \leq n!$	7m
4	66	Define normal extension and hence comment on normal extension of the following extensions $i)\ Quadratic\ extension$ and $ii)$ $[\mathbb{Q}(\sqrt[3]{2}) : \mathbb{Q}]$	7m
1	67	Define the operation of ideals and hence show that $I + J, IJ, I \cap J\ are\ ideals\ where\ Iand\ J\ are\ ideals\ of\ R$	8m
1	68	Show that every integral domain can be embedded into a field	8m
3	69	State and prove Kronecke's lemma	8m

4	70	Prove that any ring of order p^2 , where $p\ is\ a\ prime$ is commutative	8m
2	71	Define a prime element and irreducible element . Show that every prime element is irreduicible .How about the converse?Justify.	10m
4	72	Prove that any finite normal extension of F in the splitting field is of some polynomial over F. How about the converse ?Justify.	10m
4	73	Define a Perfect field.Prove that for a field $\mathbb{F}$ with $Ch\mathbb{F} = p$ is perfect if and only if $\mathbb{F} = \mathbb{F}^p\ where\ \mathbb{F}^p = \{x^p/x \in \mathbb{F}\}$.	10m
1	74	State and prove fundamental theorem of homomorphism by defining all the definitions required to prove the theorem and hence deduce that $\frac{\mathbb{Z}[x]}{(2,x)} \cong \mathbb{Z}_2$	14m
1	75	State and prove Correspondence theorem	14m
2	76	Show that every principal ideal domain is a unique factorisation domain	14m
2	77	Prove that if $R[x]$ is a UFD whenever R is a UFD.	14m
3	78	If $F \subset\ K \subset L,\ where\ F, K, L\ arefields, Prove\ that\ [L : F] = [L : K][K : F]$.Further show that $K/F\ and\ L/K$are algebraic then L/F is algebraic.	14m

Model Question Paper

Q.P Code: 16MSMTBH01

St. Philomena's College (Autonomous) Mysore

II Semester M.Sc Final Examination April - 2017

Subject: MATHEMATICS

Title: ALGEBRA - II

Time: 3 Hours **Max Marks: 70**

PART –A

Answer the following questions: 7x2=10

1. a. Find a zero divisor in the direct product ring ZXZ.
   b. Find all associates of 2 + 3*i* in Z [*i*].
   c. Factorize 2 and 5 in Z [*i*] as product of irreducible.
   d. Give an example of a finite extension which is not a normal extension.
   e. Give an example of an inseparable polynomial.
   f. If $F \leq K \leq L$ are fields such that $[K:F]=3$ and $[L:K]=7$ then find the value of $[L:F]$
   g. Is there a finite field of order 1000? Justify your answer.

PART –B

Answer the following:

2. a Define a maximal ideal. If R is a commutative ring with identity, prove that an ideal M of R is a maximal ideal if and only if R/M is a field. 8
   b In a commutative ring R shows that the set of all nilpotent elements form an ideal of R. 6

OR

3. a Prove that an integral domain can be embedded in a field. 8
   b If $\phi : R \rightarrow S$ is an onto ring homomorphism with Kernel I, prove that, $S \cong R/I$. 6

4 a Define principal ideal domain and Euclidean domain. Show that any Euclidean domain is a principal ideal domain. 8
   b Let R be a PID. Prove that any nonzero non unit in R can be written as a product of finite member of irreducibles. 6

OR

5 a If R is a unique factorization domain, show that R [x] is a unique factorization domain. 8
   b State and prove Eisenstein's Criterion for irreducibility of a polynomial over a unique factorization domain. 6

PTO

6 a If $F \leq K \leq L$ are fields with [L : K] and [K : F] are finite, show that [L : F] is finite and [L : F] = [L : K] [K : F]. **8**

b Define finite extension and algebraic extension of fields. Show that a finite extension is an algebraic extension. **6**

OR

7 a Let p(x) be irreducible over F. Show that there exists an extension K/F and $\alpha \in K$ such that $p(\alpha) = 0,\ K = F(\alpha)$ *and* $[K : F] = \deg p(x)$. **8**

b Construct a field F of order 4 and give its addition and multiplication tables. **6**

8 a State and prove primitive element theorem. **10**

b Prove that $Q(\sqrt{2}, \sqrt{3}) = Q(\sqrt{2} + \sqrt{3})$. **4**

OR

9 a Define splitting field $f(x) \in F[x]$. Show that if $\deg f(x) = n$, then there exists a splitting field K of $f(x)$ over F such that $[K : F] \leq n!$. **8**

b Define perfect field. If Ch F = b then show that F is perfect if and only if every element of F has a p^{th} root in F. **6**

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