St.Philomena's College (Autonomus), Mysore

PG Department of Mathematics

Question Bank (Revised Curriculum 2018 onwards)

Second Year - Fourth Semester (2018 -20 Batch)

Course Title (Paper Title): Advanced Graph Theory Q.P.Code-57305

Unit	S.No	Question	Marks
1	1	Define line graph and write the line graph of $K_4 - e$.	2m
1	2	Define subdivision graph with example.	2m
1	3	Define total graph with example.	2m
1	4	Draw $L(G)$ for the following degree seguences; $i).(2,2,2,2,2,2)$ $ii).(3,2,2,1,1,1)$	2m
1	5	Draw $L(G)$, $L^2(G)$ for K_4 and W_3.	2m
1	6	Is W_5 is a line graph? Justify.	2m
1	7	Is $K_4 - e$ is a line graph? Justify.	2m
1	8	Draw $L(G)$ and $L^2(G)$ of the degree sequence (3,2,2,2,1).	2m
1	9	Find adjacency eigen value of K_3 graph.	2m
2	10	Define adjacency matrix with example.	2m
2	11	Define incidence matrix with example.	2m
2	12	Define cycle matrix with example.	2m

2	13	Mention at least two differences between adjacency matrix and incidence matrix.	2m
3	14	Define distance between two vertices of a graph G with example.	2m
3	15	Define eccentricity of a vertex of a graph G with example.	2m
3	16	Define radius of a graph G with example.	2m
3	17	Define diameter of a graph G with example.	2m
3	18	Define center of a graph G with example.	2m
3	19	Define distance matrix of a graph G with example.	2m
4	20	Give an example of dominating set D such that D is common dominating set for C_5 and $\overline{C_5}$.	2m
4	21	Give an example of dominating set D such that D is common dominating set for K_5 and $\overline{K_5}$.	2m
4	22	Give an example for a minimal dominating set need not be minimum.	2m
4	23	Define minimal dominating set with example.	2m
4	24	Define domination number of a graph G with example.	2m
4	25	Find domination number of K_p and $\overline{K_p}$.	2m
4	26	Find domination number of $K_{m,n}$ and $\overline{K_{m,n}}$.	2m
4	27	Find domination number of C_n and P_n.	2m

1	28	If G is a (p, q) graph then show that $L(G)$ is a (q, q_L) graph where, $q_L = \frac{1}{2}\sum_{i=1}^{p} d_i^2 - q$.	6m
1	29	Prove that $K_{1,3}$ is not a line graph.	6m
1	30	Write all the forbidden subgraphs for line graphs.	6m
2	31	Demonstrate matrix tree theorem on $K_4 - e$.	6m
2	32	A graph G on p points is connected if and only if $(A + I)^{p-1}$ has no zero entries.	6m
2	33	Define adjacency eigen value of a graph. Find the adjacency eigen value of C_4 and W_4 graphs.	6m
3	34	Let T be a tree with $V(T) = \{v_1, v_2, \ldots, v_n\}$ $n \geq 2$ and L be Laplacian of T. then show that $\mu = 1$ is an eigen value of L with multiplicity atleast $p(T) - q(T)$.	6m
4	35	Find the minimal dominating set of C_n, $\overline{K_{m,n}}$, K_n, $K_{m,n}$, W_n, and P_n graphs.	6m
4	36	If G is a graph having p points and q lines then prove that $p - q \leq \gamma(G) \leq p - \Delta$.	6m
4	37	Prove that every non trivial connected graph G has a dominating set D whose component $V - D$ is also a dominating set.	6m
4	38	If G be any graph then show that $p - q \leq \gamma(G)$, further $\gamma(G) = p - q$ if and only if each component of G is a star.	6m

3	39	Let T be a tree with $V(T) = \{v_1, v_2, \ldots, v_n\}$, $n \geq 2$ and D be the distance matrix of T, then show that D has one positive and $n-1$ negative eigen values.	7m
3	40	Let T be a tree with $V(T) = \{v_1, v_2, \ldots, v_n\}$, D be the distance matrix of T and L be Laplacian of T. Let $\mu_1 > 0 > \mu_2 > \cdots > \mu_n$ be the eigen values of D and Let $\lambda_1 \geq \cdots \geq \lambda_{n-1} > \lambda_n = 0$ be eigen values of L, then show that $0 > \frac{-2}{\lambda_1} \geq \mu_2 \geq \frac{-2}{\lambda_2} \geq \cdots \geq \frac{-2}{\lambda_{n-1}} \geq \mu_n$.	7m
3	41	Let T be a tree with $V(T) = \{v_1, v_2, \ldots, v_n\}$, $n \geq 2$ and D be the distance matrix of T, then show that the determinant of D is given by $detD = (-1)^{n-1}(n-1)^{n-2}$.	7m
3	42	Let T be a tree with $V(T) = \{v_1, v_2, \ldots, v_n\}$, and L be Laplacian of T. Suppose $\mu > 1$ is an integer eigen value of L with u as a corresponding eigen vector, then show that the followings are hold i). μ divides n ii). No coordinate of u is zero iii). The algebraic multiplicity of μ is one.	7m
1	43	Show that the following statements are equivalent i). G is a line graph. ii). The line of G can be partitioned into complete subgraphs in such a way that no points lies in more than two of the subgraphs.	8m
1	44	A graph is the line graph of a tree if and only if it is a connected block graph in which each cut point is on exactly two blocks.	8m
1	45	The line graph of a graph G is path if and only if G is path.	8m

1	46	Let G_1 and G_2 be two connected graph with isomorphic line graph. Then show that G_1 and G_2 are isomorphic unless one is K_3 and other is $K_{1,3}$.	8m
1	47	If G_1 and G_2 are isomorphic then prove that $L(G_1)$ and $L(G_2)$ are also isomorphic.	8m
2	48	State and prove Matrix tree theorem.	8m
2	49	For any graph with incidence matrix B, show that $A(L(G)) = B^T B - 2I_q$, where B^T is a transpose of B.	8m
2	50	If $A = (a_i j)$ be the adjacency matrix of a graph G then prove that $(i,j)^{th}$ entry in A^n $[(A^n)_{ij}]$ is the number of walks of length n from v_i to v_j with example.	8m
2	51	If G has incidence matrix B and cycle matrix C then $CB^T \equiv 0(mod 2)$ where, B^T is the transpose of B.	6m
2	52	Find the labelled spanning trees on p points using matrix tree theorem.	8m
3	53	Let T be a tree with $V(T) = \{v_1, v_2, \ldots, v_n\}$ $n \geq 2$ and L be Laplacian of T. If μ is an eigen value of L then show that the algebraic multiplicity of μ is atmost $p(T) - 1$.	8m
4	54	A dominating set D is a minimal dominating set if and only if for each vertex v in$D$, one of the following condition holds; i) v is an isolated vertex of D. ii)there exist a vertex u in $V - D$ such that $N(x) \cap D = \{v\}$.	8m
4	55	If G is a graph of order n then prove that $\left\lfloor \frac{n}{1+\Delta(G)} \right\rfloor \leq \gamma(G) \leq n - \beta$.	8m

4	56	Define point covering number of a graph, then prove that for any (p, q) graph without isolated point $\gamma(G) \leq \alpha(G)$. where $\alpha(G)$ is point covering number of G.	8m
4	57	Define minimal dominating set. If G be a graph without isolated points and D is a minimal dominating set then show that $V - D$ is a dominating set.	8m
3	58	Let G be a connected graph with $V(G) = \{v_1, v_2, \ldots, v_n\}$, D be the distance matrix of G, and $G_1, G_2, \ldots, G_k$ be the blocks of G, then prove that the following are hold: i). $cofD(G) = \prod_{i=1}^{k} cofD(G_i)$ ii). $detD(G) = \sum_{i=1}^{k} detD(G_i) \prod_{j \neq i} cofD(G_i)$.	14m

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Q.P Code: 16MSMTDS05

St. Philomena's College (Autonomous) Mysore
IV Semester M.Sc. Makeup Examination September 2018
Subject: MATHEMATICS
Title: Advanced Graph Theory (SC)

Time: 3 Hours **Max. Marks: 70**

Instruction to the Candidates: ***Answer All the Questions:***

PART – A

Answer the following: **7×2=14**

1. a. Prove that there are no. 3-connected graph with 7 edges.
   b. Define a line graph with an example.
   c. Show that every planar graph contains a vertex of degree at most 5.
   d. Give two example of graphs which are both Eulerian and Hamiltonian.
   e. Define a cycle matrix with an example.
   f. Define an arboricity with an example.
   g. Mention at least two differences between adjacency matrix and incidence matrix.

PART – B

2. a. Show that: **08**
   The following statements are equivalent for a connected graph G:
   i) G is Eulerian
   ii) Every point of G has even degree
   iii) The set of lines of G can be partitioned into cycles.

   b. If $p \geq 3$ and for every pair u and v of non adjacent point ray $\deg(x) + \deg(v) \geq p$, then prove that G is Hamiltonian. **06**

OR

3. a. If G_1 and G_2 are isomorphic then prove that $L(G_1)$ and $L(G_2)$ are also isomorphic. **06**
   b. If G is a (p, q) graph where point have deg i, then prove that LCG has q points and **04**

   $$q_2 = q + \frac{1}{2}\sum d_i^2 \text{ lines}$$

   c. Write all the forbidden sub graphs for line graphs. **04**

PTO

4.	a.	Let G_1 and G_2 be connected graph with isomorphic line graphs, then G_1 and G_2 are isomorphic unless one is K_3 and other.	10
	b.	Prove that the complete graph K_{2n+1} is 2 – factorable.	04

OR

5.	a.	Prove that G is a line graph if and only if the lines of G can be portioned into complete sub group in such a way that no point lies in more than two of the sub graphs.	08
	b.	Prove that every planer graph is 5 – colourable.	06
6.	a.	For any graph G, show that $x(G) \leq 1 + \max \delta(G^1)$, where the maximum is taken over all induced sub graph G^1 of G.	06
	b.	For any graph G, show that the sum and product of x and $\bar{x}$ satisfies the inequality $2\sqrt{p} \leq x + \bar{x} \leq p+1$, $p \leq x\,\bar{x} \leq \left(\frac{p+1}{2}\right)^2$	08

OR

7.	a.	Prove that a graph is bicolorable if and only if it has not odd cycle. .	08
	b.	Show that every Peterson graph is non-hamiltonian.	06
8.	a.	State and prove matrix tree theorem.	10
	b.	If G is a graph of ordin, then prove that $\left[\frac{n}{1+\Delta G}\right] \leq \gamma(G) \leq n - \Delta(G)$	04

OR

9.	a.	Prove that G with point 'p' is tree if and only if $f(G, t) = t(t-1)^{p-1}$.	07
	b.	Show that the point graph and the line graph of a graph G are isomorphic if and only if G has at most one isolated point and K_2 is nt a component of G.	07

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