St.Philomena's College (Autonomus), Mysore

PG Department of Mathematics

Question Bank (Revised Curriculum 2018 onwards)

Second Year - Fourth Semester (2018 -20 Batch)

Course Title (Paper Title): Ordinary and Partial Differential Equations

Q.P.Code-57303

Unit	S.No	Question	Marks
1	1	Solve the IVP $y'' + xy' + xy = x + y - 1, where\ x \in\ [0\ 1]$ with the condition $y(log2) = \frac{1}{2} = y'(log2)$	2m
1	2	Solve the IVP $y'' + xy' + xy = x + y - 1, where\ x \in\ [0\ 1]$ with the condition $y(log2) = 2 = y'(log2)$	2m
1	3	Solve the IVP $y'' - y^2 + 6y = 0\ with\ y(1) = e^2\ and\ y'(1) = 3e^2$	2m
1	4	When does the uniqueness of the solution of an Initial Value problem fails?	2m
1	5	Show that $V = \{y/\ y'' + P(x)y' + Q(x)y = 0\}$ forms a vector space over real field.	2m
1	6	Define Wronskian of $y_1\ and\ y_2$ where $y_1\ ,\ y_2$ are the solutiom of second order linear ordinary differential equation and hence find $W(e^x\ , e^{-x})$	2m
1	7	Define trajectory with an example.	2m
1	8	Comment on critical points by giving suitable example	2m

1	9	Define stable critical point with an example.	2m
2	10	Define a regular singular point.Determine whether $x = 0$ is a regular singular point of $2x^2y{\prime\prime} + 7x(x+1)y' - 3y = 0$	2m
2	11	Show that x =0 is a ordinary point of $(x^2-1)y{\prime\prime} + xy' - y = 0$ but $x = 1$ is a regular singular point	2m
2	12	Define Hermite polynomial with its Generating function.	2m
2	13	Prove that $H_n'(x) = 2nH_{n-1}(x)$, where$H_n(x)$ *is Hermite polynomial*	2m
2	14	Show that $\{J_{\frac{-1}{2}}(x)\}^2 + \{J_{\frac{1}{2}}(x)\}^2 = \frac{2}{\pi x}$	2m
2	15	Show that $J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}}\sin x$	2m
2	16	Show that $J_{-\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}}\cos x$	2m
3	17	Define semilinear and quasilinear partial differential equation.	2m
3	18	Form the partial diferential equations by eliminating arbitrary constant $z = ax + by + ab$	2m
3	19	Obtain the PDE for $z = x + ax^2y^2 + b$; where $'a'$ and $'b'$ are arbitrary constants.	2m
3	20	Form the PDE by eliminating arbitrary function F for $z = xy + F(x^2+y^2)$.	2m
4	21	Form the PDE by eliminating arbitrary function F for $z = F(\frac{xy}{z})$.	2m
4	22	Form the PDE by eliminating arbitrary function F for $\phi(x+y, x-\sqrt{2}) = 0$	2m
4	23	Solve $p + q - pq = 0$ by Charpit's Method	2m
4	24	Find the complete integral of $p^2 + q^2 = x + y$	2m

4	25	Find the variables ξ *and* η which reduces to PDE $u_{xx} - x^2u_{yy} = 0$.	2m
1	26	Find a particular solution of $y'' + 4y = \tan 2x$.	4m
1	27	By Wronskian to prove that two solutions of second order linear homogeneous equation are linearly independent if they have common zero in the inteval .	4m
1	28	Show that $x^2y'' + xy\prime + (x^2 - p^2)y = 0$; where $x \neq 0$ has infinitely many zeroes.	4m
1	30	Comment on the nature of the critical points by its eigen value of a dynamical system with two constraints.	4m
2	31	Prove that $2nJ_n(x) = xJ_{n-1}(x) + J_{n+1}(x)$	4m
2	32	Prove that $xJ'_n = -nJ_n(x) + xJ_{n-1}(x)$	4m
3	33	Solve $(z-y)p + (x-z)q = y - x$.	4m
2	34	Prove that $xJ'_n(x) = nJ_n(x) - xJ_{(n+1)}(x)$	5m
3	35	Solve $\left(\frac{y^2z}{x}\right)p + xzq = y^2$	5m
3	36	Solve $p + 3q = 5z + tan(y - 3x)$	5m
3	37	Find the which intersects the surfaces of the system $z(x+y) = c(3z+1)$ orthogonally and which passes through the the circle $x^2 + y^2 = 1;\ z = 1$.	5m
3	38	Find the surface which is orthogonal to the parameter system $z = cxy(x^2 + y^2)$ and which passes through the hyperbola $x^2 - y^2 = a^2$ and $z = 0$.	5m

1	39	Explain the method to find the second solution using a known solution for second order linear homogeneous differential equation.	6m
1	40	Find the particular solution of $y'' + y = \cot^2 x$ and $y'' + y = \cot 2x$	6m
1	41	Show the critical points of $\frac{dx}{dt} = x\ and\ \frac{dy}{dt} = -y\ is\ a\ saddle\ point$	6m
2	42	Prove the following $P_n(x) = P'_{n+1}(x) - 2xP'_n(x) + P'_{n-1}(x)$	6m
4	43	Comment on classification of PDE in the variables and hence classify the PDE $u_{xx} + u_{yy} + u_{zz} = 0$.	6m
1	44	Define Wronskian and show that wronskian is identicaly zero or never zero.	7m
1	45	Show that $the\ solution\ y_1(x),\ y_2(x)\ of\ y'' + p(x)y' + Q(x)y = 0$ are linearly dependent if and only if $W(y_1, y_2) = 0$	7m
1	46	Verify that $y = e^x\ is\ a\ known\ solution\ of\ xy'' - (2x+1)y' + (x+1)y = 0$.Hence find the general solution.	7m
1	47	Find the general solution of $(x^2-1)y'' - 2xy' + 2y = (x^2-1)^2$	7m
2	48	State and prove the Rodrigue's formula for Legendre Polynomial	7m
2	49	Show that $\int_{-1}^{1} P_m(x)P_n(x) = \begin{cases} \frac{2}{2n+1}, & \text{if } m = n \\ 0, & \text{otherwise} \end{cases}$	7m
3	50	Find the complete integral of $px + qy = pq$; $q \neq 0$. $(7m)$	7m
3	51	Solve by Charpit's method $px + qy = pq; where\ p = \frac{\partial z}{\partial x}\ \ q = \frac{\partial z}{\partial y}$.	7m

3	52	Use the Adomian Method to solve IVP $u_{tt} = u_{xx}, 0 < x < \pi; t > 0$ Boundary Condition $u(0,t) = 0$ $u(\pi,t) = 0$ initial conditions $u(x,0) = 0, u_t(x,0) = sinx;$	7m
3	53	Use the Decomposition method and the noise term phenomenon to solve the following inhomogeneous PDE $u_x + u_y = (1+x)e^y; u(0,y) = 0$, $u(x,0) = x$	7m
4	54	Solve $u_{tt} - c^2 u_{xx} = 0 \ with \ u(x,0) = f(x); u_t(x,0) = g(x); -\infty < x < \infty; t > 0$.	7m
4	55	Solve the following homogeneous heat equation by Adomian method, $PDE \ u_t = u_{xx}; 0 < x < \pi, t > 0$ Boundary Condition $u(0,t) = 0; t \geq 0; u(\pi,t) = 0; t \geq 0;$ Initial Condition $u(x,0) = sinx$	7m
4	56	Use modified decomposition method to solve $u_x + u_y = (1+x)e^y; u(0,y) = 0; u(x,0) = x$	7m
4	57	Use the modified decomposition method to solve Ricatti's Differential Equation $y' = Ax^2 + y^2; \quad y(0) = 0$	7m
4	58	Use the Adomian Decomposition method to solve IVP PDE ;$u_t = u_{xx}, 0 < x < \pi; t > 0$ Boundary Condition $u(0,t) = e^{-t}; t \geq 0$ $u(\pi,t) = \pi - e^{-t}; t \geq 0$ initial conditions $u(x,0) = x + cosx;$	7m

1	59	Show that $y_1(x) = \frac{cosx}{\sqrt{x}}$ is a known solution of $x^2y'' + xy' + (x^2 - \frac{1}{4})y = 0$. Hence solve the Initial value problem $y'' + \frac{1}{x}y' + (1 - \frac{1}{4x^2})y = 0;\ x \neq 0\ with\ the\ condition\ y(\frac{\pi}{2}) = \frac{2}{\pi}\ and\ y'(\frac{\pi}{2}) = -\frac{2}{\pi}(1 + \frac{1}{\pi})$.	8m
1	60	Explain the method of finding the solution by transforming a linear ordinary differential equations into normal form	8m
2	61	Explain the power series method for Bessels diferential equation	8m
4	62	Reduce $e^{2x}u_{xx} + u_{yy} + 2e^{x+y}u_{xy} + e^{2y}u_{yy} = c\ into\ canonical\ form.$	8m
1	63	State and prove Strum's separation theorem	10m
1	64	If $u(x)$ is a nontrival solution of $u'' + q(x)u = 0, \forall x > 0$ and $\int_1^{\infty} q(x)dx = \infty$; then prove that q(x) has infinitely many zeroes on positive $x - axis$	10m
1	65	State and prove Picard's theorem for existence and uniqueness of initial value problem.	14m
2	66	Explain the general method to find the power series solution of second order linear differential equation and hence solve the Airy's equation $y'' + xy = 0$	14m

Q.P Code: 16MSMTBS04

St. Philomena's College (Autonomous) Mysore

II Semester M.Sc. C3 Component - Final Examination April - 2017

Subject: MATHEMATICS

Title: ORDINARY AND PARTIAL DIFFERENTIAL EQUATIONS (SC)

Time: 3 Hours **Max Marks: 70**

PART –A

Answer the following questions: **7x2=14**

1. a. Define a Stable Critical point with an example.

   b. When do the given two solutions are linearly dependent?

   c. If $z = f(x^2 - y) + g(x^2 - y)$, show that $\frac{\partial^2 z}{\partial x^2} - 4x^2 \frac{\partial^2 z}{\partial y^2} = \frac{1}{x}\frac{\partial z}{\partial x}$.

   d. Show that $y_1 = \cos x + \sin x$, $y_2 = \cos x - \sin x$ are two linearly independent solution of $y^{11} + y = 0$.

   e. Write the normal form of a Legendre polynomial and find $q(x)$ in it.

   f. State Strum Comparison theorem.

   g. Write the Parseval's equation.

PART –B

Answer the following:

2. State and prove Picard's theorem for existence and uniqueness of solution. **14**

OR

3. a State and prove Sturm's Separation theorem. **8**

   b Write and general solution of initial Value problem $\frac{d^2 y}{dx^2} - 4\frac{dy}{dx} + 4y = 0$, $y(0) = 1$ and $y^1(0) = 2$. **6**

PTO

			10
4.	a	Find the general solution of the Gauss hypergeometric equation $x(1-x)y'' + [c-(a+b+1)x]y' - ab\ y = 0$ in the form of Frobenious series solution valid in $0 < x < 1$.	4
	b	Show that Zero's of $J_n(x)$ are simple.	
		OR	8
5	a	Show that $(n+1)P_{n+1}(x) = (2n+1)\ x P_n(x) - n P_{n-1}(x)$ and $P_n(x) = P_{n+1}^1(x) - 2x P_n^1(x) + P_{n-1}^1(x)$.	6
	b	Show that $\int_{-1}^{1} P_m(x)P_n(x)\ dx = \begin{cases} 0 & m \neq n \\ \dfrac{1}{2n+1} & m = n \end{cases}$.	7
6.	a	Describe the Green's functions method for solving a standard Strum – Liouville problem.	7
	b	Solve by Greens function method $y'' + y = f(x),\ x \in \left[0, \frac{\pi}{2}\right],\ y'(0) = 0,\ y'\left(\frac{\pi}{2}\right) = 0$.	
		OR	
7	a	Reduce $\frac{\partial^2 z}{\partial x^2} + 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = 0$ to Cannonical form and hence solve it.	5
	b	Derive one dimensional wave equation for a string.	5
	c	Prove that $\int_{-\infty}^{\infty} f(x)\ \delta(x-t)\,dx = f(t)$.	4
8	a	Show that all the eigen values of Strum – Liouville's problem are real.	8
	b	Show that the set of functions $\left\{\sin\left(\frac{n\pi x}{c}\right)\right\}$ $n = 1, 2, \ldots..$ is orthogonal in (0, c).	
		OR	
9	a	Let $f(\theta)$ be any given continuous function of θ with period 2π. Prove that the Dirichlet problem $u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta} = 0,\ 0 \le r < 1\ W(1, \theta) = f(\theta),\ -\pi \le \theta \le \pi$ has a solution given by Poisson integral formula.	10
	b	State the necessary condition for the Validity of the Fourier series expansion of an integrable function.	4

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