St.Philomena's College (Autonomus), Mysore

PG Department of Mathematics

Question Bank (Revised Curriculum 2018 onwards)

Second Year - Fourth Semester (2018 -20 Batch)

Course Title (Paper Title): Theory Of Numbers Q.P.Code-57304

Unit	S.No	Question	Marks
1	1	If P_n is the n^{th} prime, then prove that $P_n \leq 2^{2^{n-1}}$	2m
1	2	Prove that the only prime p for which $3p+1$ is a perfect square is $p=5$.	2m
1	3	Prove that the only prime of the form n^3-1 is 7.	2m
1	4	State Dirichlet's theorem an hence deduce that there are infinitely many primes whose last three digits are 999.	2m
1	5	If P_n is the n^{th} prime, then prove that $N = P_1 \cdot P_2 \cdots P_n + 1$ is never a perfect square for any n.	2m
1	6	If $n > 2$, then prove that there exist a prime p satisfying $n < p < n!$.	2m
1	7	If $p \neq 5$ is an odd prime, then prove that p^2-1 or p^2+1 is divisible by 10.	2m
1	8	For $n \geq 2$, show that the last digit of $F_n$is 7.	2m
1	9	Find the successor of $\frac{2}{111}$ in $\mathfrak{F}_{257}$.	2m
2	10	Show that $\displaystyle\sum_{\frac{p}{q}\in\mathfrak{F}_n} \frac{p}{q} = \frac{\lvert\mathfrak{F}_n\rvert}{2}$.	2m
2	11	Prove that $F_0 \cdot F_1 \cdots F_{n-1} = F_n - 2$.	2m

2	12	Find the number of elements in $\mathfrak{F}_n$.	2m
2	13	Show that Fermat Number F_n for $n = 5$ is composite.	2m
2	14	Define Farey sequence.	2m
2	15	Show that $\sqrt[m]{N}$ is irrational unless $N = n^m$ for any n.	2m
2	16	Show that $\varphi(n) = \sum_{d\|n} \mu(d) \cdot \frac{n}{d}$	2m
2	17	Define an Arithmetical function with an example.	2m
2	18	State Mobius inversion Formula.	2m
2	19	Define Dirichlet product of two arithematical functions.	2m
2	20	Show that Λ is neither a multiplicative function, nor a completely multiplicative function.	2m
3	21	Define a multiplicative function with an example.	2m
3	22	Given two completely multiplicative functions f and g, show that $f = g$ if and only if $f_p(x) = g_p(x)$ for all primes p.	2m
3	23	Define Bell series.	2m
3	24	Show that $\sum_{d\|n} \Lambda(d) = \log n$	2m
3	25	Define Mangoldt function.	2m
3	26	If a'_n is the n^{th} complete quotient of a continued fraction $[a_0, a_1, a_2, \cdots, a_N]$, then find the integral part of a'_{N-1}.	2m
3	27	If $\frac{p_n}{q_n}, \frac{p_{n-1}}{q_{n-1}}$ are the n^{th} and $(n-1)^{th}$ convergents of a continued fraction $[a_0, a_1, a_2, \cdot\cdot\cdot, a_N]$, then show that $\frac{p_n}{q_n} - \frac{p_{n-1}}{q_{n-1}} = \frac{(-1)^{n-1}}{q_n \cdot q_{n-1}}$ $\mathbb{F}$	2m

3	28	Express $\frac{29}{10}$ as a finite simple continued fraction.	2m
4	29	If $[a_0, a_1, a_2, \cdots, a_N]$ is a finite simple continued fraction, then show that $q_n \geq q_{n-1}$ for $n \geq 1$.	2m
4	30	If $[a_0, a_1, a_2, \cdots, a_N]$ is a finite simple continued fraction, then show that $q_n \geq n$ for $n \geq 1$.	2m
4	31	If $\frac{p_n}{q_n}$ is the n^{th} convergent of a finite simple contiued fraction, then show that $\frac{p_n}{q_n}$ is an irreducible rational number.	2m
4	32	Obtain the continued fraction for $x = \frac{1+\sqrt{5}}{2}$.	2m
4	33	If $[a_0; a_1, a_2, a_3, \cdots]$ is an infinte simple continued fraction such that it converges to x , then show that x is an irrational number.	2m
4	34	Define equivalent numbers. Show that any two integers are equivalent.	2m
4	35	Show that the relation $\xi \sim \eta$ is an equivalence relation.	2m
4	36	Define periodic continued fraction.	2m
1	37	If p is an odd prime, $a, b \in \mathbb{Z}^+$ such that $(a, p) = (b, p) = (a, b) = 1$ and $p \mid a^2 + b^2$, then show that $p \equiv 1(\ mod\ 4\ )$.	3m
1	38	Find the values of $n \geq 1$ for which is $n! + (n+1)! + (n+2)!$ a perfect square.	3m
1	39	Prove that $\frac{1}{P_1} + \frac{1}{P_2} + \cdots + \frac{1}{P_n}$ is never an integer.	4m
1	40	Show that $(F_n, F_m) = 1$ for $n \neq m$.	4m
4	41	Show that $\sqrt{2}$ is never equivalent to $\sqrt{3}$.	4m

1	42	Show that there are infinitely many primes of the form $6n+5$.	7m
1	43	If P_n is the n^{th} prime, then prove that $P_n \sim n \log_e n$ for large n.	7m
2	44	If $\varphi(n)$ is Euler's function, then show that $\sum_{d\|n} \varphi(d) = n$	7m
2	45	Show that $\varphi(n) = n \sum_{d\|n} \frac{\mu(d)}{d}$.	7m
2	46	For any two intgers m, n, show that $\varphi(m \cdot n) = \varphi(m) \cdot \varphi(n) \cdot \frac{d}{\varphi(d)}$, where $d = (m, n)$.	7m
2	47	Show that if $a\|b$ then $\varphi(a)\|\varphi(b)$.	7m
2	48	If both g and $f * g$ are multiplicative, then show that f is also multiplicative.	7m
2	49	If f is multiplicative, then show that f is completely multiplicative if and only if $f^{-1}(n) = \mu(n) \cdot f(n) \quad \forall \quad n \geq 1$.	7m
2	50	Show that $\sum_{d\|n} \Lambda(d) = \log n$ for $n \geq 1$.	7m
2	51	Show that for $n \geq 1$, $\Lambda(n) = \sum_{d\|n} \mu(d) \cdot \log \frac{n}{d} = -\sum_{d\|n} \mu(d) \cdot \log d$.	7m
3	52	Prove that every odd convergent is greater than any even convergent in a continued fraction. .	7m
3	53	If $[a_0, a_1, a_2, \cdots, a_N]$ is a finite continued fraction, then show that $p_n = a_n p_{n-1} + p_{n-2}, \quad q_n = a_n q_{n-1} + q_{n-2}, \quad n \geq 2$.	7m
3	54	Show that the even convergents increase strictly with n while the odd convergents decrease strictly in a continued fraction.	7m

3	55	Find the value of the continued fraction $[-2; 1, 2, 5, 7, 4, 1, 6]$ If $[a_0, a_1, a_2, \cdots, a_N] = [b_0, b_1, b_2, \cdots, b_M]$ and $a_N > 1,\ b_N > 1$ then prove that N=M and $a_i = b_i\ \ \forall i,\ 0 \le i \le N$.	7m
3	56	Show that if x is representable by a simple continued fraction with an odd (even) number of convergents, then it is also representable by one with an even (odd) number.	7m
3	57	Solve the linear diophantine equation $172x + 50y = 500$.	7m
3	58	Given any rational number , show that it can be expressed as a finite simple continued fraction.	7m
3	59	Show that any infinite simple continued fraction $[a_0, a_1, a_2, \cdots]$ converges.	7m
3	60	If $[a_0, a_1, a_2, \cdots,] = [b_0, b_1, b_2, \cdots,]$ then show that $a_i = b_i\ \ \forall i,\ \ i = 0, 1, 2, \cdots$	7m
4	61	Show that given any irrational number ξ has an infinite simple continued fraction reprezentation.	7m
4	62	Show that any two rationals are equivalent.	7m
4	63	Show that any two irrational numbers ξ and η are equivalent if and only if $\xi = [a_0, a_1, a_2, \cdots, a_m, c_0, c_1, c_2, \cdots], \eta = [b_0, b_1, b_2, \cdots, b_n, c_0, c_1, c_2, \cdots]$, the sequence of quotients in ξ after the m^{th} being the same as the sequence in η after the n^{th}.	7m
4	64	Show that a periodic continued fraction is a quadratic surd.	7m
1	65	Show that $\sum_{i=1}^{\infty} \frac{1}{P_i}$ is a divergent series, where P_i is the i^{th} prime.	8m

1	66	If $\pi(x)$ is the prime counting function, then show that $\log logx \leq \pi(x)$ for $x \geq 2$.	8m
2	67	Show that e^y is irrational for $y \neq 0, y \in \mathbb{Q}$.	8m
2	68	If f is an arithmetical function with $f(1) \neq 0$, then show that there is a unique arithmetical function f^{-1} called the Dirichlet inverse of f such that $f * f^{-1} = f^{-1} * f = I$ where $I(n) = \left[\frac{1}{n}\right] = \begin{cases} 1, & if \quad n = 1 \\ 0 & if \quad n > 1 \end{cases}$.	8m
1	69	State and prove Pepin's test.	10m
4	70	Show that the continued fraction which represents a quadratic surd is periodic.	10m

(8)

Model Question Paper Q.P Code:50633

St. Philomena's College (Autonomous) Mysore
III Semester M.Sc. Make-up Examination August - 2019

Subject: MATHEMATICS

Title: Theory of Numbers

...ne: 3 Hours **Max Marks: 70**

...ruction to the Candidates: ***Answer All the questions. All questions carry equal marks.***

PART – A

Answer the following: **7×2=14**

a. If P_n is n^{th} prime then show that $\frac{1}{p_1}+\frac{1}{p_2}+\ldots\ldots\frac{1}{p_n}$ is never an integer.

b. Show that $\frac{\log 2}{\log 10}$ is an irrational number.

c. Show that $\phi(n)$ is even for $n \geq 3$.

d. Prove that there are infinitely many integer 'n' such that $\phi(n) \neq 3k$.

e. Show that $2^{10}(2^{11}-1)$ is not a perfect number.

f. If $n=2^{K-1}(2^K-1)$, $K \geq 2$ in a perfect number prove that $\pi_{d/n} d = n^K$.

g. Show that any two rationals are equivalent.

PART – B

a. State and prove fundamental theorem of Arithmetic. 08

b. Show that there are infinitely many primes of the form $8n+5$. 06

OR

a. Show that π^2 is irrational. 08

b. Show that the series $\sum_{p\ prime} \frac{1}{p}$ is divergent. 06

a. If $n \geq 1$, show that $\sum_{d/n} \mu(d) = \begin{cases} 1 & in \quad n=1 \\ 0 & if \quad n>1 \end{cases}$. 04

b. If $n \geq 1$ then prove that $\sum_{d/n} \phi(d) = n$. 06

c. Show that $\frac{\phi(n)}{n} = \sum_{d/n} \frac{\mu(d)}{d}$, where $n \in Z^+$. 04

OR

PTO

5. a. Show that Dirchlet product of two multiplicative function is multiplicative.

b. If both f and $f * g$ are multiplicative then prove that f is also multiplicative.

6. a. If $2^K - 1$ is prime $(K > 1)$ then prove that $n = 2^{K-1}(2^K - 1)$ is perfect and every even perfect number is or this form. How about the converse? Justify.

b. For an even perfect number $n > 6$, show that the sum of digits of n is congruent to 1 mod 9.

OR

7. a. Show that $U_{mn} = U_{m-1}U_n + U_m U_{n+1}$ *for* $m \geq 2, n \geq 1$.

b. Prove that $U_1^2 + U_2^2 + ... + U_n^2 = U_n U_{n+1}$.

c. Show that a perfect square cannot be a perfect number.

8. a. Prove that every odd convergent of a continued fraction is greater than any even convergent.

b. Prove that every infinite simple continued fraction converges.

OR

9. a. Show that every irrational number can be expressed uniquely as an infinite simple continued fraction.

b. Prove that two rational numbers ξ and η are equivalent if and only if $\xi = [a_0, a_1, a_2, ...a_m, c_0, c_1, c_2, ...]$ and $\eta = [b_0, b_1, b_2, ...b_{v,}\ c_0, c_1, c_2, ...]$