St.Philomena's College (Autonomus), Mysore

PG Department of Mathematics

Question Bank (Revised Curriculum 2018 onwards)

Second Year - Fourth Semester (2018 -20 Batch)

Course Title (Paper Title): Topology-II Q.P.Code-57302

Unit	S.No	Question	Marks
1	1	Define First countable space with an example.	2m
1	2	Define Second countable space with an example.	2m
1	3	Define Lindelof space with an example.	2m
1	4	Define Normal space with an example.	2m
1	5	Is the product of normal spaces normal? Justify.	2m
1	6	Is the product of Lindelof spaces a Lindelof space. Justify.	2m
1	7	Is every first countable space second countable space? Justify.	2m
1	8	Consider the normal space $\mathbb{R}$ and the closed subsets $A = [-2, -1]$ and $B = [2, 3]$ of $\mathbb{R}$. Give a Urysohn function $f : \mathbb{R} \to [0, 1]$ such that $f(A) = \{0\}$ and $f(B) = \{1\}$.	2m
1	9	State Tietze's extension theorem.	2m
2	10	Give an example of a completely regular space that is not normal.	2m
2	11	Give an example of a completely regular space that is not normal.	2m
1	12	State the Imbedding theorem.	2m

2	13	Define an m-manifold.	2m
2	14	State Tychonoff theorem.	2m
2	15	Give an example of a collection of subsets of $\mathbb{R}$ that is locally finite.	2m
2	16	When do you say that a collection B of subsets of X is countably locally finite open refinement of a collection of subsets of X?	2m
3	17	Is a paracompact space always compact? Justify.	2m
3	18	Define a paracompact space.	2m
3	19	Is every subspace of a paracompact space paracompact? Justify.	2m
3	20	Define a covering map.	2m
4	21	Define a simply connected space.	2m
4	22	Give a covering map and the covering space of S^1, the unit circle in the plane $\mathbb{R}^2$.	2m
4	23	Give two paths in $\mathbb{R}^2 - \{0\}$ having the same initial points which are not path homotopic.	2m
4	24	Define essential and inessential maps.	2m
4	25	State Fundamental Theorem of Algebra.	2m
1	26	Show that a Hausdorff space need not be regular.	6m
2	27	Prove that a compact m- manifold X can be imbedded in $\mathbb{R}^N$ for some $N \in \mathbb{Z}^+$.	6m
2	28	Show that a completely regular space is regular..	6m

3	29	Prove that every metrizable space has a countably locally finite basis for its topology.	6m
3	30	Prove that $\mathbb{R}$ is paracompact.	6m
4	31	Prove that the relation "$f \simeq_p g$" is an equivalence relation.	6m
4	32	Prove that the map $p : \mathbb{R} \to S^1$ given by the equation $p(x) = (\cos 2\pi x, \sin 2\pi x)$ is a covering map.	6m
1	33	Let X be first countable, $A \subset X$ and $x \in X$. Then prove that $x \in \overline{A}$ if and only if there is a sequence of points of A converging to x.	7m
1	34	Show that if X is Lindelof and Y is compact then $X \times Y$ is Lindelof.	7m
1	35	Define a normal space. Prove that a regular space with a countable basis is normal.	7m
1	36	Prove that every compact Hausdorff space is normal.	7m
1	37	Prove that $\mathbb{R}_l \times \mathbb{R}_l$ is not Lindelof.	7m
1	38	Show that a subspace of a separable space need not be separable.	7m
2	39	Prove that a product of completely regular spaces is completely regular.	7m
2	40	Prove that Tietze's extension theorem implies Urysohn's lemma.	7m
4	41	Define an inessential map. Let $h : X \to Y$ be an inessential map. Prove that h is the zero homomorphism	7m
4	42	State and prove fundamental theorem of algebra	7m

1	43	Define a regular space. Let X be a topological space in which one point sets are closed. Prove that X is regular if and only if given a point x of X and a neighbourhood U of x, there exist a neighbourhood V of x such that $\overline{V} \subset U$.	8m
2	44	Let A and B be closed disjoint subsets of the normal space X. Prove that there exists a continuous function $f : X \to [0,1]$ such that $f^{-\prime}(\{0\}) = A$ and $f^{-\prime}(B) = \{1\}$ if and only if A is a $G_\delta$set in X.	8m
2	45	State and prove Urysohn's metrization theorem.	8m
3	46	Prove that every paracompact space is normal.	8m
3	47	Let X be a regular space. If every open covering of X has a refinement that is an open covering of X and is countably locally finite, then show that the open covering has a refinement that covers X and is locally finite.	8m
4	48	If X is path connected and $x_0, x_1 \in X$, prove that $\pi_1(X, x_0)$ is isomorphic to $\pi_1(X, x_1)$.	8m
4	49	Prove that the fundamental group of the circle is infinite cyclic.	8m
1	50	Prove that $\mathbb{R}_\ell$ is a first countable, Lindeloff, separable space but not a second countable space.	14m
2	51	State and prove Urysohn's lemma.	14m
2	52	State and prove Tietze's extension theorem.	14m

3	53	State and prove Tychonoff's theorem.	14m

Model Question Paper

Q.P Code: 16MSMTDH02

St. Philomena's College (Autonomous) Mysore

IV Semester M.Sc. Mathematics Final Examination : April 2018

Subject: MATHEMATICS

Title: Topology - II

Time: 3 Hours

Max. Marks: 70

Instruction to the Candidates: *Answer All the Questions*:

PART – A

Answer the following: 7×2=14

1. a. Show that R_l is not second countable.

   b. If X_α, for each α, is nonempty and $\underset{\alpha}{\pi} X_\alpha$ is Hausdorff, show that each X_α is Hausdorff.

   c. If in a topological space X, two disjoint closed sets A and B are separated by a continuous function, show that there exist disjoint open sets containing A and B.

   d. Show that the Tietze extensiion theorem implies the Urysohn lemma.

   e. If A is a locally finite collection of subsets of a space X, prove that $\overline{\bigcup_{A \in A} A} = \bigcup_{A \in A} \overline{A}$.

   f. Show that every regular Lindelof space is paracompact.

   g. If $h, h' : X \to Y$ are homotopic and $k, k' : Y \to Z$ are homotopic, then show that $k \circ h$ *and* $k' \circ h'$ are homotopic.

PART – B

2. a. Define first and second countable spaces. Let X be first countable, $A \subset X$ and $x \in X$. Prove that $x \in \overline{A}$ is and only if there is a sequence of points of A converging to x. **07**

   b. Prove that every compact metrizable space X is second countable. **07**

OR

3. a. Define regular and normal spaces. Prove that every normal space is regular. How about the converse? Justify. **07**

   b. Prove that every compact Hausdorff space is normal. **07**

PTO

4.		State and prove the Urysohn lemma.	14

OR

5.	a.	State and prove the Urysohn metrization theorem.	07
	b.	If X is a compact m-manifold, then prove that X can be imbedded in R^N for some positive integer N.	07
6.	a.	If X is metrizable, prove that every open covering of X has a countably locally finite open refinement that covers X.	10
	b.	Prove that every closed subspace of a paracompact space is paracompact.	04

OR

7.		Prove that every metrizable space is paracompact.	14
8.	a.	Let the operation '*' on path – homotopy classes be defined by: $[f]*[g]=[f*g]$, where $f(1)=g(0)$. Prove that the operation '*' is well-defined and associative. Further show that there	08
	b.	If X is path connected and $x_0, x_1 \in X$, prove that $\pi_1(X, x_0)$ is isomorphic to $\pi_1(X, x_1)$.	06

OR

9.	a.	Prove that the fundamental group of S^1 is infinite cyclic.	10
	b.	If $h: X \to Y$ is inessential, prove that h_* is the zero homomorphism.	04

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