St.Philomena's College (Autonomus), Mysore

PG Department of Mathematics

Question Bank (Revised Curriculum 2018 onwards)

Second Year - Third Semester (2019 -21 Batch)

Course Title (Paper Title):Elements Of Functional Analysis

Q.P.Code-57201

Unit	S.No	Question	Marks
1	1	Define a complete metric space with an example.	2m
1	2	Show that $\mathbb{R}^n$ and $\mathbb{C}^n$ are complete metric spaces.	2m
1	3	Define a normed linear space	2m
1	4	Prove that every normed linear space is metric space	2m
1	5	Define a contraction map and a fixed point	2m
1	6	Define first category and second category metric spaces	2m
2	7	Show that ${l_n}^p(\mathbb{R})$ is a complete metric space	2m
2	8	Define a separable space with an example	2m
2	9	Find the dense subspace of $l^p(\mathbb{R})$	2m
2	10	Show that every dense subspace of $l^\infty(\mathbb{R})$ is uncountably infinite	2m
3	11	Is Bolzano Weirstrass property is true in general metric space? Justify	2m
3	12	Is Cantor's intersection property is true i general metric space? Justify	2m
3	13	Show that the boundedness in $\mathbb{R}$ implies total boundedness	2m

3	14	Define total boundedness of a set with an example	2m
3	15	Define linear operator and bounded linear operator	2m
4	16	Define a poset with an example	2m
4	17	Define Banach space with n example	2m
4	18	Define an open map	2m
4	19	Show that $l_2^{\infty}(\mathbb{R})$ is an Hilbert space	2m
4	20	In a Hilbert space define Orthogonality and Orthogonal Compliment	2m
1	21	Define complete metric space.Give two examples	4m
1	22	Define isometry with an example. Is every isometry is continuous? Justify .	4m
1	23	Define Nowhere Dense set and Everywhere dense subset with an example?	4m
2	24	Define first category and second category metric spaces.	4m
4	25	Prove that $\|\|x+y\|\|^2 - \|\|x-y\|\|^2 + i\|\|x+iy\|\|^2 - \|\|x-iy\|\|^2 = 4xy$	4m
4	26	In a Hilbert space show that inner product space is continuous.	6m
1	27	Show that every closed subspace of a complete metric space is complete.	7m
1	28	Define a normed linear space .Show that every normed linear space is a complete metric space	7m
2	29	Prove that a Normed linear space is complete if and only if every absolutely convergent series in X is convergent	7m

3	30	Show that the linear space $B(N, N')$ over $\mathbb{F}$ is a normed linear space with respect to $\|\|T\|\| = Sup\{ \ \|\|T(x)\|\|; \|\|x\|\| \leq 1 \ \}$	7m
4	31	In a Hilbert space show that inner product is continous.	7m
4	32	If M is a closed linear subspace of a Hilbert Space H then prove that $H = M \oplus M^{\perp}$	7m
4	33	In a Hilbert Space H , Define orthonormal set and prove Bessel's inequality.	7m
3	34	Define a an inner product space and Normed linear space . If $(H, (.,.))$ is an inner product space then prove that it is a Normed linear space	8m
1	35	Show that $B^*((x), d))$ is a complete metric space	10m
1	37	State and prove Metric completion theorem	10m
1	38	State and prove Banach fixed point theorem	10m
1	39	State and prove Baire's Category theorem.	10m
4	40	In a Hilbert space prove the Paralleogram law and Pythogoras theorem	10m
1	41	Prove Picards theorem as a consequence of Banach Fixed Point theorem	14m
2	42	Show that the following is equialent for the metric space (X, d) a) X is Compact b) X is Sequentially Compact c) X is complete and totally bounded	14m

2	43	If $T : N \to N'$ is a linear operator from an normed linear space N to a normed linear space N' then show that the following are equivalent a) T is continuous linear operator. b) T is continuous at $x = 0$. c) T is bounded linear operator. d) If $S = \{x \in N; \|\|x\|\| \leq 1\}$ is the closed unit sphere in N then $T(S)$ is bounded in N'	14m
3	44	State and prove Hahn Banach theorem for any Real linear space .	14m
3	45	State and prove Hahn Banach theorem for any Complex linear space .	14m
3	46	State and prove Open mapping theorem by explaining the definitions that are needed to prove the theorem.	14m
3	47	State and prove Closed Graph theorem by explaining the definitions that are needed to prove the theorem.	14m
3	48	State and prove Principle of uniform boundedness by explaining the definitions that are needed to prove the result.	14m

Let H be a Hilbert space such that $S \subset H$ then prove the following results

a) $S \cap S^{\perp} = \{0\}$

4 49 b) $S^{\perp}$ is a closed subspace of H. 14m

c) $S_1 \subseteq S_2$ that implies $S_2^{\perp} \subseteq S_1^{\perp}$.

d) $S \subseteq S^{\perp\perp}$

If e_i is an orthonormal set in a Hilbert Space H then prove that the following is equivalent.

a) e_i is complete.

4 50 14m

b) $x \perp e_i \implies x = 0$

c) $x \in H \implies \quad x = \sum (x, e_i) e_i$.

d) $x \in H \implies \quad ||x||^2 = \sum ||(x, e_i)||^2$

Q.P Code:50631

St. Philomena's College (Autonomous) Mysore
III Semester M.Sc. Make-up Examination July - 2019
Subject: MATHEMATICS
Title: Elements of Functional Analysis

Time: 3 Hours

Max Marks: 70

Instruction to the Candidates: *Answer All the questions. All questions carry equal marks.*

PART –A

Answer the following: 7×2=14

1. a. Show that a Cauchy sequence in a metric space need not necessarily be convergent.
   b. Show that every isometry in a metric space is continuous.
   c. State Hahn-Banach theorem for a normed linear space.
   d. What do you mean by metric completion?
   e. State principle of uniform boundedness.
   f. In a Hilbert space state and prove parallelogram law.
   g. State and prove Bessel's inequality for a Hilbert space with finite orthonormal set.

PART - B

7+7

2. a. State and prove Banach fixed point theorem.
   b. State and prove Cantor's intersection theorem.

OR

3. State and prove Ascoli-Arzella's theorem. 14
4. State and prove Stone Weierstrass theorem for real case. 14

OR

7+7

5. a. State and prove Hahn-Banach theorem for complex linear space.
   b. If $\frac{1}{p}+\frac{1}{q}=1,\ p\neq\infty$ prove that $\overset{*}{l}_p = l_q$.
6. State and prove open mapping theorem. 14

OR

7. a. State and prove uniform boundedness theorem. 07
   b. State and prove existence of isometric isomorphism from $B(N,N)$ into $B(N^*,N^*)$. 07

PTO

8. a. Define a complete orthonormal set in a Hilbert space. As an example show that 7+7

$\left(\frac{\sin nx}{\sqrt{\pi}}, \frac{\cos nx}{\sqrt{\pi}}\right)$ is the classical complete orthonormal set in $L^2[0, 2\pi]$.

b. If H is a Hilbert space and let $\{e_i\}$ be an orthonormal set in H, prove the following are equivalent:

i) $\{e_i\}$ is complete,

ii) $x \perp \{e_i\} \Rightarrow x = 0$

iii) $x \in H \Rightarrow x = \Sigma(x, e_i) e_i$

iv) $x \in H \Rightarrow \|x\|^2 = \Sigma |(x, e_i)|^2$

OR

9. a. If M and N are closed linear subspace of a Hilbert space H and if $M \perp N$, prove that $M + N$ is also a closed subspace of H. 4+10

b. State and prove Riesz – representation theorem on conjugate space of a Hilbert space.

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