St.Philomena's College (Autonomus), Mysore

PG Department of Mathematics

Question Bank (Revised Curriculum 2018 onwards)

First Year - Third Semester (2019 -21 Batch)

Course Title (Paper Title): Topology-I Q.P.Code-57202

Unit	**Sl.No**	**Question**	**Marks**
1	1	Show that the topologies of $\mathbb{R}_l$ and $\mathbb{R}_k$ are not comparable.	2m
1	2	Let A and B denote subsets of a space X. Prove that $\overline{A \cup B} = \overline{A} \cup \overline{B}$.	2m
1	3	Find the boundary and the interior of the subsets $A = \{x \times y / y = 0\}$.	2m
1	4	Show that if $X = \{a, b, c\}$ the collection of all one point subsets of X is a basis for the discrete topology on X.	2m
1	5	Prove that one point sets are closed in Haussdorff space.	2m
1	6	If $\mathcal{B}$ is a basis for the topology of X and $Y \subset X$. Show that the collection $\mathcal{B}_Y = \{B \cap Y / B \in \mathcal{B}\}$ is a basis for the subspace topology on Y.	2m
1	7	If A is a subset of a topological space. Prove that $\overline{A} = IntA \cup BdA$.	2m
1	8	If A is a subsets of a space X. Prove taht $IntA$ and BdA are disjoint.	2m
1	9	Find the set of all limit points of $A = \left\{\frac{1}{n} : n \in \mathbf{Z}^+\right\}$.	2m
1	10	Define a T_1 space. Is T_1 space is a Haussdorff? Justify.	2m
1	11	Define subbasis of topological space with an example.	2m
2	12	Prove that the composite map of two continuous maps is continuous.	2m

2	13	Suppose that $f : X \to Y$ is continuous. $A \subset X$, and $x \in X$. If x is a limit point of A, is it necessarily true that $f(x)$ a limit poit of $f(A)$? Justify.	2m
2	14	If A is a connected subspace of a space X and $A \subset B \subset \overline{A}$. Prove that B is also conneted.	2m
2	15	Show that the unit ball B^n is path connected in $\mathbf{R}^n$.	2m
2	16	Let $Y = [0, 1) \cup (1, 2]$ be a subspace of $\mathbf{R}$ with standard topology. Check wheather Y is connnected or not.	2m
2	17	Define a locally path connected space.	2m
2	18	Show that $X = \{0\} \cup \left\{ \frac{1}{n} : n \in \mathbf{Z}^+ \right\}$ of $\mathbf{R}$ is compact.	2m
2	19	Show that every closed subsets of a compact space is compact.	2m
2	20	Define Homeomorphism with an example.	2m
2	21	Define a quotient map with an example.	2m
3	22	Define a linear Continuom.	2m
3	23	Define a path connected space.	2m
3	24	Define a locally path connected space.	2m
3	25	Is the space $\mathbf{R}_l$ is connected? Why?	2m
3	26	Show that a component in a topological space is connected.	2m
4	27	Is the set $(0, 1)$ compact in $\mathbf{R}$? Why?	2m
4	28	Show that every closed subset of a compact space is compact.	2m

4	29	Show that $(0, 1]$ is not compact in the space $\mathbf{R}$.	2m
4	30	Show that a bijective continuous mapping from a compact space to a Hausdorff space is a homeomorhism.	2m
4	31	Give an example of a topological space that is not compact but is locall compact.	2m
4	32	Define Lebesque number.	2m
4	33	Justify with an example that a limit point compact space need not be compact.	2m
4	34	Is compact subset of a topological space is closed? Justify.	2m
1	35	Prove that the lower limit topology on $\mathbf{R}$ is strictly finer than the standard topology on $\mathbf{R}$.	4m
1	36	Let $\mathcal{S}$ be a subbasis for a topology on X. Then prove that the collection $\mathcal{B}$ of all finite intersection of elements of $\mathcal{S}$ form a basis for a topology on X.	4m
1	37	Let X be a topological space, $A \subset X$. Then prove that $\overline{A} = A \cup A'$.	4m
1	38	Prove that a subspace of a Hausdorff space is Hausdorff space.	4m
1	39	Let X be a set and τ_f be the collection of all subsets U of X such that $X - U$ is either finite or all of X. Prove that τ_f is a topology on X.	6m

1	40	Let $\mathcal{B}$ and $\mathcal{B}'$ be bases for topologies τ and τ' respectively. Prove that τ' is finer than τ if and only if for each $x \in X$ and each element $B' \in \mathcal{B}'$ such that $x \in B' \subset B$.	6m
1	41	Let X and Y be two topological spaces and $$\mathcal{S} = \left\{\pi_1^{-1}(U) : U \textit{ is open in } X\right\} \cup \left\{\pi_2^{-1}(V) : V \textit{ is open in } Y\right\}.$$ Then prove that $\mathcal{S}$ is a subasis for the product topology on $X \times Y$.	6m
1	42	Prove that the product of two Hausdorff space is Hausdorff.	6m
1	43	Let X be a topological space. let A be a subset of X. Then prove that $x \in \bar{A}$ if and only if every open set U containg x intersects A.	6m
1	44	Let X be a topological space and Y be a subspace of X. Prove that a subset A of Y is closed in Y if and only if $A = C \cap Y$ where C is some closed set in X.	8m
1	45	Let X be a T_1- space, A be a subset of X and $x \in X$. Then prove that $x \in A'$ if and only if every neighbourhood of x contains infinitely many points of A.	8m
1	46	Show that X is a Hausdorff space if and only if the diagonal $\triangle = \{x \times x / x \in X\}$ is closed in $X \times X$	8m

2	47	Let $f : X \to Y$ be a map. Then prove that the following are equivalent. 1. f is continuous . 2. for every subset A of X, $f(\bar{A}) \subset \overline{f(A)}$. 3. for every closed subset B of Y, $f^{-1}(B)$ is closed in X.	8m
2	48	Prove that $\mathbf{R}^W$ is metrizable in the product topology.	14m
2	49	Prove that arbitraty product of Hausdorff spaces is Hasudorff.	8m
2	50	State and prove Pasting Lemma.	6m
2	51	Let $f : A \to X \times Y$ be given by the equation $f(a) = (f_1(a), f_2(a))$, where $f_1 : A \to X$ and $f_2 : A \to Y$. Prove that f is continuous if and only if f_1 and f_2 are continuous.	6m
2	52	Prove that $\mathbf{R}^n$ is metrizable.	8m
2	53	Let $f_n : X \to Y$ be a sequence of continuous functions from a topological space into a metric space. Show that if $f_n \to f$ uniformly, then f is continuous.	6m
2	54	Let $f : X \to Y$ be a mappig from a metrizable space X into a topological spave Y. Prove that f is continuous if and only if for every sequence $\{x_n\}$ in X converging to x, the sequence $\{f(x_n)\}$ converges to $f(x)$ in Y.	8m
2	55	State and prove sequence lemma.	6m
3	56	Prove that a space X is connected if and only if its only subsets that are both open and closed are $\emptyset$ and X itself.	6m

3	57	Prove that if a collection of connected subsets have a point in common, then the union of elements of the collection is connected.	6m
3	58	Prove that continuous image of a connected space is connected.	6m
3	59	Let L be a linear continum in the order topology. Then prove that every iterval or a ray in L is connected.	8m
3	60	Prove that $I \times I = [0,1] \times [0,1]$ is a linear continum in the dictionary order.	6m
3	61	Let X be a topological space. Prove that the path components of X are disjoint, path connected subsets whose union equals X and each path connected set intersects only one of them.	8m
3	62	Prove that a space is locally connected if and only if has a basis consisting of connected sets	8m
3	63	Let X be a topological space. Then prove that each path component of X is contained in a component of X. Also, show that if X is locally path connected, then they are equal.	6m
3	64	Prove that the product of connected spaces in the product topoloy is connected.	14m
4	65	Let Y be a subspace of a topological space X. Prove that Y is compact in subspace topology if and only if every covering of Y by sets open in X has a finite sub-collection that covers Y.	8m

4	66	State and prove Tube lemma.	6m
4	67	Prove that the product of finitely many compact space is compact.	8m
4	68	Let X be a simply ordered set with the least upper bound property. Then prove that every closed interval in X is compact.	8m
4	69	Prove taht a subset of $\mathbf{R}^n$ is compact if and only if it is closed and bounded.	8m
4	70	Let X be a Hausdorff space. then prove that X is locally compact if and only if for each $x \in X$, each open set U of x, there is an open set V containing x such that $\overline{V}$ is compact and $\overline{V} \subset U$.	8m
4	71	Let X be a compact space and Y be an order set in the order topology. If $f : X \to Y$ is continuous, then prove that there exist $c, d \in X$ such that $f(c) \leq f(x) \leq f(d) \;\; \forall \;\; x \; \in X$.	6m
4	72	Let X be a metrizable space. Prove that the following are equivalent: 1. X is compact. 2. X is limit point compact. 3. X is sequentially compact.	14m

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Q.P Code:50632

St. Philomena's College (Autonomous) Mysore
III Semester M.Sc. Make-up Examination August - 2019
Subject: MATHEMATICS
Title: TOPOLOGY - I

Time: 3 Hours **Max Marks: 70**

Instruction to the Candidates: *Answer All the questions. All questions carry equal marks.*

PART –A

Answer the following: 7×2=14

1. a. Define an order topology and give an example.

   b. Prove that the collection $S = \{\pi_1^{-1}(\cup) / \cup \text{ is open in } X\} \cup \{\pi_2^{-1}(V) / V \text{ is open in } Y\}$ is a sub basis for the product topology on $X \times Y$.

   c. Is every open set containing $x \in X$, intersects $E \subseteq X$ at infinitely many points, where x is a limit point of E? Justify.

   d. Determine the closure of the following subsets of R^2

      i) $\{1/n \times 0 \ / \ n \in N\}$

      ii) $\{x \times 0 \ / \ 0 < x < 1\}$

   e. Define one-point compactification and give an example.

   f. Prove that a path connected space is connected.

   g. Define homeomorphism and give an example.

PART – B

2. a. If β and β^1 is basis for the topologies T and T^1 respectively on X. Prove that the following are equivalent: 06

      i) T^1 is finer than T

      ii) for each $x \in X$ and each basis element $B \in \beta$ containing x, there is a basis element $B^1 \in \beta^1$ such that $x \in B^1 \subseteq B$

   b. If X is a toplogical space and C is a collection of open sets of X such that for each $x \in X$ and each open set U of X containing x, there is an element c of C such that $x \in c \subseteq U$ then prove that C is a basis for the topology of X. 08

OR

3. a. Prove that the lower limit topology on R is strictly finer than the standard topology. 04

PTO

b. If Y is a subspace of X, prove that a set A is closed in Y if and only if it equals the intersection of a closed set of X with Y. **06**

c. If B is a basis for a topology T on X, prove that T equals the collection of all unions of elements of B. **04**

4. a. Prove that $x \in \overline{A}$ is and only if every open set U containing x intersects A. **04**

b. If X and Y are topological spaces and $f : X \to Y$ is a function, prove that the following are equivalent: **06**

i) f is continuous

ii) for every subset A of X $f(\overline{A}) \subseteq \overline{f(A)}$

iii) for every closet subset B in Y, the set $f^{-1}(B)$ is closed in X.

c. State and prove Pasting lemma. **04**

OR

5. a. State and prove sequence lemma, further prove that R^{ω} with box topology is not metrizable. **07**

b. If $f : A \to \pi_{\alpha \in J} x_\alpha$ is given by the equation $f(a) = (f_\alpha(a))_{\alpha \in J}$ where $f_\alpha : A \to x_\alpha$ for each α and $\pi_{\alpha \in J} x_\alpha$ have the product topology, prove that the function f is continuous if and only if each function f_α is continuous. Further prove that the above statement is not true if $\pi_{\alpha \in J} x_\alpha$ has the box topology. **07**

6. a. If A is a connected subset of X, and $A \subseteq B \subseteq \overline{A}$ prove that B is also connected. **04**

b. Define Linear continuum. If L is a linear continuum in the order topology, prove that L is connect. **10**

OR

7. a. Prove that the union of collections of connected sets that have a point common is connected. **04**

b. State the prove Intermediate value theorem. **05**

c. Prove that finite cartition product of connected set is connected. **05**

8. a. If Y is a compact subset of the Hausdarff space X and $x_0 \notin Y$, prove that there exist disjoint open sets U and V of X containing x_0 and Y respectively. **05**

b. Prove that product of finitely many compact sets is compact. **09**

OR

9. a. If X is a metrizable space, prove that the following are equivalent: **07**

i) X is compact

ii) X is limit point compact

iii) X is sequentially compact

b. Prove that the continuous image of a compact set is compact. Further if $f : X \to Y$ is objective continuous function, where X is compact and Y is Hausdorff, prove that f is a homeomorphism. **07**