St.Philomena's College (Autonomus), Mysore

PG Department of Mathematics

Question Bank (Revised Curriculum 2018 onwards)

Second Year - Third Semester (2019 -21 Batch)

Course Title (Paper Title): Mathematical Computation

(Interdisciplinary paper)

Q.P.Code-96553

Unit	S.No	Question	Marks
1	1	Define boolean algebra.	2m
1	2	Define partial order set.	2m
1	3	Define lattice.	2m
1	4	Define complemented lattice.	2m
1	5	Define distributive lattice.	2m
1	6	Define order and size of a graph. Find order and size of complete graph.	2m
1	7	Draw all graphs with four points.	2m
1	8	Define multi-graph and pseudograph with example.	2m
1	9	What is the maximum number of lines in a graph with p points? Justify.	2m
1	10	Draw two different (5,5) graph.	2m

1	11	Draw a graph with point representing the numbers $1, 2, 3, ..., 10$ in which two points are adjacent if and only if they have a common divisor greater than one.	2m
1	12	Define degree of a point in a graph. Draw all graphs on $p = 3$ points.	2m
1	13	Define minimum degree of a graph with example.	2m
1	14	Define Maximum degree of a graph with example.	2m
1	15	Define regular graph. Draw all regular graph with four points.	2m
1	16	Define cubic graph. Draw all cubic graph with eight points.	2m
1	17	Define complete graph with example.	2m
1	18	Define complement and self complement of a graph with example.	2m
1	19	Let G be a (p,q) graph then prove that for $v \in V(G)$ $deg_{\overline{G}}v = p - 1 - deg_G v$.	2m
1	20	Show that a (p,q) graph is a complete graph if and only if $q = \frac{p(p-1)}{2}$.	2m
1	21	Find the complement of K_p and show that K_p is a (p-1)regular graph.	2m
1	22	Define spanning subgraph of a graph. Draw all spanning subgraph of K_3.	2m
1	23	Define induced subgraph of a graph. Draw two induced subgraph of K_5.	2m
1	24	Define complete bipartite graph. Give an example of a bipartite graph which is regular.	2m
1	25	Define distance between two points with example.	2m

1	26	Define union of two graphs with example.	2m
1	27	Define join of two graphs with example.	2m
1	28	Define product of two graphs with example.	2m
1	29	Define composition of two graphs with example.	2m
1	30	Define tree. Draw all trees with four points.	2m
1	31	Draw all trees with seven points and $\Delta(T) \geq 4$.	2m
1	32	Prove that every cubic graph has even number of points.	4m
1	33	Prove that every graph with at least two points contains two points of the same degree.	5m
1	34	Let G be a (p,q) graph then prove that $\delta \leq \frac{2q}{p} \leq \Delta$. Does there exist a 3- regular graph with six points? If so construct the graph.	5m
1	35	Show that the relation $\geq$ is a partial ordering on the set of integers.	6m
1	36	Find the compliment of the following boolean expression, i)$x(y'z' + yz)$. ii)$ab' + ac + b'c$.	6m
1	37	State and prove First theorem of graph theory.	7m
1	38	Prove that in any graph the number of point of odd degree is even.	7m
1	39	Prove that any self complementary graph has 4n or 4n+1 points.	7m
1	40	Prove that if G is regular then $\overline{G}$ is also regular.	7m
1	41	Prove that for any graph with six points G or $\overline{G}$ contains a triangle.	7m

1	42	Prove that a non trivial graph is bipartite if and only if all its cycles are even.	7m
1	43	If $(L, \vee, \wedge)$ is a complemented distributive lattice then prove that $(a \vee b)' = a' \wedge b'$ and $(a \wedge b)' = a' \vee b'$.	8m
1	44	Let $B = \{1, 5, 7, 35\}$ be the set of positive integers and operations $+$ and $\cdot$ are defined as follows: a+b=lcm(a,b) and $a \cdot b = gcd(a, b)\ \forall a, b \in B$. An unary operation $'$ on B is defined as $a' = \frac{35}{a}\ \forall a \in B$. Show that $(B, +, \cdot, ')$ is a boolean algebra.	8m

Unit	S.No	Question	Marks
2	1	Geometrically interpret 2x2 matrix.	2m
2	2	Define Skew Symmetric matrix.What will be the determinant of a skew symmetric matrix of odd order?Justify.	2m
2	3	Is the product of two lower traingular matrix is lower traingular? Justify	2m
2	4	By trail and error find an example of 2x2 matrix such that $A^2 = -I$,A has only real entries.	2m
2	5	Find a 2×2 matrix $B^2 = 0$ although $B \neq 0$	2m
2	6	Find a 2×2 matrix $EF = 0$ although no entries of E of F are zero.	2m
2	7	Verify $A(\theta_1 + \theta_2) = A(\theta_1)A(\theta_2)$ for $A(\theta)= \begin{pmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{pmatrix}$	2m
2	8	Compute the products $$\begin{bmatrix} 4 & 0 & 1 \\ 0 & 1 & 0 \\ 4 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix} \text{ and } \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 5 \\ -2 \\ 3 \end{bmatrix} \text{ and } \begin{bmatrix} 2 & 0 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$ For the third one, draw the column vectors $(2,1)$ and $(0,3)$. Multiplying by $(1,1)$ just adds the vectors (do it graphically).	5m
2	9	Working a column at a time, compute the products $$\begin{bmatrix} 4 & 1 \\ 5 & 1 \\ 6 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \end{bmatrix} \text{ and } \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \text{ and } \begin{bmatrix} 4 & 3 \\ 6 & 6 \\ 8 & 9 \end{bmatrix} \begin{bmatrix} \frac{1}{2} \\ \frac{1}{3} \end{bmatrix}.$$	5m

2	10	Find two inner products and a matrix product: $\begin{bmatrix} 1 & -2 & 7 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \\ 7 \end{bmatrix}$ and $\begin{bmatrix} 1 & -2 & 7 \end{bmatrix} \begin{bmatrix} 3 \\ 5 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ -2 \\ 7 \end{bmatrix} \begin{bmatrix} 3 & 5 & 1 \end{bmatrix}$.	5m
2	11	Multiply Ax to find a solution vector x to the system $Ax =$ *zero vector*. Can you find more solutions to $Ax = 0$? $Ax = \begin{bmatrix} 3 & -6 & 0 \\ 0 & 2 & -2 \\ 1 & -1 & -1 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$.	5m
2	12	If A and B are square matrices, show that $I - BA$ is invertible if $I - AB$ is invertible.	5m
2	13	Prove that A is invertible if $a \neq 0$ and $a \neq b$ (find the pivots and A^{-1}): $A = \begin{bmatrix} a & b & b \\ a & a & b \\ a & a & a \end{bmatrix}$.	5m
2	14	Verify that $(AB)^{\mathrm{T}}$ equals $B^{\mathrm{T}}A^{\mathrm{T}}$ but those are different from $A^{\mathrm{T}}B^{\mathrm{T}}$: $A = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$ $AB = \begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix}$.	5m
2	15	Find the reduced row echelon forms R and the rank of these matrices: (a) The 3 by 4 matrix of all 1s. (b) The 4 by 4 matrix with $a_{ij} = (-1)^{ij}$. (c) The 3 by 4 matrix with $a_{ij} = (-1)^j$.	5m
2	16	Use the cofactor matrix C to invert these symmetric matrices: $A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{bmatrix}$.	5m
2	17	Find the solution of $3x + 2y = 7$ and $4x + 3y = 11$	5m

2	18	Prove that determinant of a 2×2 matrix changes its sign when two rows are exchanged.What will happen if the rows are equal ?	4m
2	19	Prove that for 3×3 matrix $\|A\| = \|A^T\|$.	4m
2	20	Define a linear transformation.Give an example of a linear transformation from $\mathbb{R}^3$ to $\mathbb{R}^2$.	4m
2	21	Is every linear transformation is a matrix ? Justify.	4m

Q.P Code: 96553

St. Philomena's College (Autonomous) Mysore

III Semester M.Sc. Final Examination : December - 2019

Subject: Interdisciplinary

Title: Mathematical Computation (ID)

Time: 3 Hours **Max Marks: 70**

PART - A

Answer all questions: **10×2=20**

1. a. Define a complemented lattice with an example.

   b. Define a Boolean lattice with an example.

   c. Define induced subgraph.

   d. Show that in any graph G, odd degree vertices are even in number.

   e. Write four non-isomorphic graph with same radius and diameter.

   f. Define singular matrix. Interpret it geometrically.

   g. Write the syntax for "if……else" statement.

   h. Write the syntax for "while loop" statement.

   i. Describe the purpose of the "close" and "close all" command in MATLAB.

   j. Define scripts and functions in MATLAB.

PART – B

Answer any THREE from the following: **3×10=30**

2. a. Define Bounded lattice. Show that for a bounded lattice $(A, \leq)$ **05**

   i) $0 \vee x = x = x \vee 0$

   ii) $0 \wedge x = 0 = x \wedge 0$

   b. Prove that the complement is unique in a distributive lattice. **05**

3. a. State and prove Demorgan's law for Boolean lattice. **07**

   b. Using Boolean Algebraic approach, show the validity of the following syllogistic argument: **03**

   All X's are Y's

   All Y's are Z's

4. a. Solve the system of equations: **05**

$$x + y - 3z = 4$$
$$4x + 3y - 2z = 2$$
$$2x + y - 3z = 1$$

PTO

I - 30
T - 62
92

b. Find the inverse of $A = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ 4 & 9 & 1 \end{pmatrix}$. **05**

5. a. If A and B are invertible matrices prove that $AB = BA$. **04**

b. Solve for x ; $\begin{vmatrix} x & 6 & -1 \\ 2 & -3x & x-3 \\ -3 & 2x & x+2 \end{vmatrix} = 0$. **06**

6. a. Prove that for any tree has a center consisting of either a point or line. **04**

b. Prove that a graph G is bipartite if and only if G contains no odd cycle in it. **06**

PART - C

Answer any TWO of the following: **2×10=20**

7. a. Write the script file and syntax to multiply the following matrics: **05**

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 1 & 2 & 5 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & 1 & 3 \\ 5 & 0 & -2 \\ 2 & 3 & -1 \end{bmatrix}.$$

b. Write the syntax to write a scalar matrix of order 4 × 4 with entries 1 and a diagonal matrix of order 4 × 4. **05**

8. a. Explain the purpose of the following commands: **05**
   - cd
   - diary
   - load
   - what
   - format

b. Explain any four format function used for numeric display. **05**

9. a. List any four plotting commands and explain their purpose. **05**

b. Write a program to need data from a text file followed by storing and displaying the data. **05**

10. a. Write syntax to find the element wise operations such as addition, subtraction and multiplication: **05**

$A = [5, 23, 11, 7]$; $B = magic(4)$

b. Write the syntax to write the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 7 & 9 & 1 \\ 4 & 6 & 3 \end{bmatrix}$. **05**

****