St.Philomena's College (Autonomus), Mysore
PG Department of Mathematics
Question Bank (Revised Curriculum 2020 onwards)
First Year - First Semester (2021 -23 Batch)
Course Title (Paper Title): Group Theory

Unit	S.No	Question	Marks
1	1	Define order of an element. If $O(a) = 25\ for\ a \in G$ then find $O(a^{20})$.	2m
1	2	If $H \lneqq G$ and $K \lneqq G$ where $O(G) < \infty$ such that $(O(H), O(K)) = 1$ then find the elements of $H \cap K$.	2m
1	3	If $a \in G$ such that $O(a) = m$ then $O(a^k) = \dfrac{m}{(m,k)}$	2m
1	4	For any $a \in G$ show that $f_a : G \to G$ defined by $f_a(x) = axa^{-1}$ is an automorphism of G	2m
2	5	If $O(a) = 20$ find the $O(xa^{12}x^{-1})$	2m
2	6	If f is an automorphism of a group G and $N \trianglelefteq G$ then show that $f(N) \trianglelefteq G$	2m
2	7	Show that every subgroup of an abelian group is normal	2m
2	8	If G is a group and $H \leq G$ of index 2 in G then show that $H \trianglelefteq G$	2m
3	9	If $G =< a >$ is a cyclic group of order 120, then find the order of a^{50}	2m
3	10	As a consequence of Fundamental theorem of Homomorphism show that G/G=0 and G/0=G	2m
3	11	Show that S_n $n \geq 3$ has a trivial center	2m
3	12	Find the order of Sylow 2 subgroup in S_9	2m
3	13	Find the order of Sylow 3 subgroup in S_7	2m
4	14	Define a p-sylow subgroup. Find the order of Sylow 3 subgroup in S_{13}	2m
4	15	Determine all the automorphisms of the additive group $\mathbb{Z}$	2m
4	16	Determine all the permutation in S_4 which are conjugate to (1234)	2m
4	17	Prove or Disprove "If every proper subgroup of a group is normal then G is abelian"	2m
4	18	Show that $SL_n(\mathbb{R}) \trianglelefteq GL_n(\mathbb{R})$	2m

1	19	Let G be a cyclic group of order n. Show that G has a unique subgroup of order d for every divisor d of n. Determine all subgroups of $\mathbb{Z}_{15}$	10m
1	20	Define normal subgroup.Show that a subgroup H of a group G is normal if and only if a right coset of H in G is a left coset of H in G. Find all the normal subgroups of S_6.	10m
1	21	State and prove Lagrange's theorem for finite groups.Justify the converse with an example. Does the converse is valid for cyclic and abelian groups.If it is valid does the subgroups are unique? justify.	10m
1	22	Define the commutator subgroup of a group G' of G. Show that G' is a normal subgroup of$G$ and G/G' is Abelian.Determine the commutator subgroup of S_3	10m
1	23	State and prove fundamental theorem of group homomorphism. Define a n copy map with an example.	10m
2	24	State and prove first isomorphism theorem. Prove that any finite cyclic group of order n has $\phi(n)$ generators.	10m
2	25	State and prove Second isomorphism theorem.Classify upto isomorphism of all groups of order 45.	10m
2	26	If $H \leq GX = G/H$, where $H \trianglelefteq G$ .Then prove that there is a group homomorphism $\xi : G \to Sym(X)$ with the kernel K is the largest normal subgroup of G which is contained in H.	10m
2	27	Define an inner automorphism of a group G. Prove that the group of inner automorphism $I(G) \cong G/Z(G)$. Find the $I(G)$ of Z_p.	10m
2	28	State and prove Cayley's theorem.	10m
3	29	Define the external direct product of Groups. Find the order of elements of $\mathbb{Z}_2 \times \mathbb{Z}_3$ .Prove that $\mathbb{Z}_m \times \mathbb{Z}_n$ is cyclic if and only if $(m, n) = 1$	10m

3	30	State and prove fundamental theorem on finite abelian group. Find the number of abelian groups of order 169.	10m
3	31	Find the number of elements of order 10 in $\mathbb{Z}_{10} \times \mathbb{Z}_{15}$.Find the number of elements of order 3 in $S_3 \times \mathbb{Z}_6$.If $G/Z(G)$ is cyclic then G is abelian.Can $O(G/Z(G)) = p\ , where\ p - prime$ whenever G is abelian	10m
3	32	Define the Dihedral group D_{2n}. Find the center of D_{2n}. Find the number of conjugates of $\sigma = (12)(345)$ in S_7	10m
3	33	Prove or disprove the following statements a) $(\mathbb{Q}^*, .)$ is cyclic b) $(\mathbb{Q}, +)$ is cyclic c) $(\mathbb{R}, +)$ is cyclic	10m
3	34	For a finite group G, if $H \leq G$ and $K \leq G$ prove that HK is a subgoup of G if and only if $HK = KH$. Deduce the result $O(HK) = \dfrac{O(H)O(K)}{O(H \cap K)}$	10m
3	35	Define the center of a group $Z(G)$, prove that $Z(G) \trianglelefteq G$. If $G/Z(G)$ is cyclic prove that G is abelian . Further if G is finite non abelian group then prove that $O(Z(G) \leq \frac{1}{4}O(G)$.	10m
3	37	Prove that any subgroup of a cyclic group is cyclic. If every proper subgroup of a group is cyclic does the group is cyclic? Justify	10m
4	38	If $'a'$ has infinite order , then all distinct powers of $'a'$ are distinct in G and in particular the cyclic subgroup of $< a >$ is infinite. If G has no nontrivial subgroups show that G is of prime order.	10m
3	39	If $a, b \in G$ such that $ab = ba$. If $O(a) = n, O(b) = m$, such that $(m, n) = 1$ then prove that $O(ab) = mn$. If $H\ K$ are subgroups of a group G of indices m and n and if $(m, n) = 1$. Show that $H \cap K$ is of index mn	. 10m

1	40	Derive the class equation for a finite group. Deduce that any group of order p^n where $p-prime$ and $n \geq 1$ has nontrivial center. Show that if $n = 2$ G is abelian.	10m
3	41	State and prove first Sylow's theorem	10m
2	42	Show that every permutation $\sigma \in S_n$can be expressed as a product of transposition. Hence define A_n and show that for $n \geq 3$ A_n is the only subgroup of S_n of index 2.	10m
4	43	If G is a cyclic group of order n then prove that G has $\phi(n)$ number of automorphisms. Hence prove that a group of order $n > 2$ has an automorphism which is not an inner automorphism	10m
4	44	Prove that disjoint permutations commute. If $H \lneq G$ and $O(G) < \infty$ such that $O(G) \nmid [G:H]!$. Prove that G is not simple.	10m
4	45	Define the Signature map . Show that for $n \geq 2$ the signature map is an homomorphism. Find the signature of $\sigma = (1234)$ in S_4	10m

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St. Philomena's College (Autonomous), Mysore

M. Sc-Mathematics (CBCS)

I/II/III/IV- Semester Examination: 2020-21

Subject:

Time: 3 Hours Max Marks: 70

Sl. No			Marks
Section – A (MCQ)			
1	a		1
	b		1
	c		1
	d		1
Section – B			
2	a		2
	b		2
	c		2
Section – C Answer any three from the following			
	3		3x10=30
	4		
	5		
	6		
Section – D Answer any three from the following			
	7		3x10=30
	8		
	9		
	10		