St.Philomena's College (Autonomus), Mysuru
PG Department of Mathematics
Question Bank (Revised Curriculum 2020 onwards)
First Year - First Semester (2021 -23 Batch)
Course Title (Paper Title): Real Analysis-I Q.P.Code-

Unit	Sl.No	Question	Marks
1	1	Find the least upper bound and greatest lower bound of the set $S = \{x \in \mathbb{R}/3x^2 - 10x + 3 < 0\}$.	2m
1	2	Show that between any two real numbers there exists an irrational numbers.	2m
1	3	Let $\{a_n\}$ be a sequence of real numbers where $a_n = \frac{n!}{n^n}$. Show that $\lim_{n\to\infty} a_n = 0$.	2m
1	4	Find $\lim_{n\to\infty} \frac{e + e^{1/2} + e^{1/3} + \cdots, +e^{1/n}}{n}$.	2m
2	5	Prove that the sequence $\left\{\frac{(-1)^n}{n}\right\}$ is Cauchy sequence.	2m
2	6	Find the limit superior and limit inferior of the sequence $\{x_n\}$ where $x_n = \begin{cases} 1 + \frac{1}{n}, \text{ if } n \text{ is even} \\ -1 - \frac{1}{n}, \text{ if } n \text{ is odd.} \end{cases}$	2m
2	7	Find the limit superior and limit inferior of the sequence $\{x_n\}$ where $x_n = (-1)^n \left(1 + \frac{1}{n}\right)^n$.	2m
2	8	Find the limit superior and limit inferior of the sequence $\{x_n\}$ where $x_n = (-1)^n\, n^{\frac{1}{n}}$.	2m
2	9	Show that every convergent sequence is bounded. What about the converse.	2m
2	10	Prove that $\lim_{n\to\infty} \left[\left(\frac{2}{1}\right)^1 \left(\frac{3}{2}\right)^2 \left(\frac{4}{3}\right)^3 \cdots, \left(\frac{n+1}{n}\right)^n\right]^{1/n} = e$.	2m
2	11	Show that $\lim_{n\to\infty} \frac{1}{n}\left(1 + 2^{\frac{1}{2}} + 3^{\frac{1}{3}} + \cdots, +n^{\frac{1}{n}}\right) = 1$.	2m
2	12	Prove that $\lim_{n\to\infty} \left(\frac{n^n}{n!}\right)^{\frac{1}{n}} = e$.	2m
3	13	Show that the series $\sum_{n=1}^{\infty} \left(\frac{3}{4}\right)^{n-1}$ converges to 4.	2m

3	14	If a series $\sum a_n$ is convergent, then show that $\lim_{n\to\infty} a_n = 0$. Is the converse true?	2m
3	15	Test the convergence of the series $\sum_{n=1}^{\infty} \sin\left(\frac{1}{n}\right)$	2m
3	16	Show that the series $\sum_{n=1}^{\infty}\left(\frac{1}{n^2}\right)^{1/n}$ is divergent.	2m
3	17	Discuss the convergence or divergence of the series $\sum \frac{n!}{n^n}$.	2m
3	18	Test the convergence of the series $\sum\left(\frac{n}{n+1}\right)^{n^2}$	2m
3	19	Find the radius of convergence of the power series $\sum_{n=0}^{\infty} \frac{2^{n+1}}{n!} z^n$.	2m
3	20	Find the radius of convergence of the power series $\sum_{n=0}^{\infty}\left\{\left(1+\frac{1}{n}\right)\left(1+\frac{2}{n}\right)\cdots,\left(1+\frac{n}{n}\right)\right\} z^n$.	2m
3	21	Discuss the convergence of the series $\sum_{n=0}^{\infty} \frac{(2n)!(3n)!}{n!(4n)!}$.	2m
3	22	Test the convergence of the series $\sum_{n=1}^{\infty} \frac{\arctan n}{1+n^2}$.	2m
3	23	Test the convergence of the series $\sum_{n=1}^{\infty} \frac{e^{1/n}}{n^2}$.	2m
3	24	Discuss the convergence of the series $\sum_{n=0}^{\infty} \frac{(2n)!(3n)!}{n!(4n)!}$.	2m
4	25	What derangement of the series $1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\cdots$, will reduce its sum to $\frac{1}{2}\log 2$.	2m
4	26	Show that the infinite products $\prod_{n=1}^{\infty}\left(1+\frac{1}{n}\right)$ is divergent.	2m
4	27	Find the radius of convergence of the series $\sum \frac{2^n}{n!} z^n$.	2m
4	28	Test the convergence and absolute convergence of the series $\sum_{n=2}^{\infty} \frac{(-1)^{n-1}}{\log n}$	2m
4	29	Use Leibnitz test to show that $\sum_{n=1}^{\infty} \frac{(-1)^n(n+5)}{n(n+1)}$ is convergent.	2m
4	30	Show that the infinite product $\prod_{n=1}^{\infty}\left(1-\frac{1}{(n+1)^2}\right)$ converges to $\frac{1}{2}$.	2m
4	31	If the product $\prod_{n=1}^{\infty}(1+a_n)$ is convergent. Show that $\lim_{n\to\infty} a_n = 0$.	2m
4	32	Find the sum of the series $1+\frac{1}{3}+\frac{1}{5}-\frac{1}{2}+\frac{1}{7}+\frac{1}{9}+\frac{1}{11}-\frac{1}{4}+\cdots$.	2m
4	33	Find the sum of the series $1+\frac{1}{3}+\frac{1}{5}-\frac{1}{2}-\frac{1}{4}+\frac{1}{7}+\frac{1}{9}+\frac{1}{11}-\frac{1}{6}-\frac{1}{8}+\cdots$.	2m

4	34	Investigate what derangement of the series $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \cdots$ will reduse its sum to zero.	2m
1	35	State and prove power mean inequality of real numbers.	6m
1	36	State and prove Archimedean property of real numbers.	4m
1	37	State and prove Hölder's inequality.	6m
1	38	Prove that $\mathbb{Q}$ is dense in $\mathbb{R}$.	4m
1	39	Show that for any real number $x > 0$ and for every integer $n > 0$ there exists a unique positive real number y such that $y^n = x$.	10m
1	40	If A, G and H are respectively Arithmetic, geometric and harmonic means of positive reals $a_1, a_2, \ldots, a_n$ then show that $H \leq G \leq A$.	7m
1	41	Prove that set of all complex number is not an ordered field.	3m
1	42	State least upper bound property of real numbers. Show that the set of rational numbers $\mathbb{Q}$ does not have this property.	6m
1	43	State and prove Minkowski's inequality.	4m
1	44	State and prove genralized arithmetic geometric mean inequality.	10m
1	45	Find the supremum and infimum of the set $S = \{x \in \mathbb{R}/(x-a)(x-b)(x-c)(x-d) < 0, a, b, c, d \in \mathbb{R}, a < b < c < d.\}$.	4m
2	46	If $\{x_n\}$ is a sequence of nonnegative reals and if $\lim_{n\to\infty} x_n = x$ then show that $x \geq 0$.	4m
2	47	Prove that every bounded sequence $\{x_n\}$ of real numbers contains a convergent subsequence.	4m
2	48	Show that set of all sub sequential limits of a real sequence $\{x_n\}$ is a closed subset of $\mathbb{R}$.	4m
2	49	Suppose $\{x_n\}$ is monotonically increasing sequence. Show that $\{x_n\}$ is convergent if and only if it is bounded.	6m
2	50	State and prove Cesaro's limit theorem.	6m

2	51	Show that a real sequence $\{x_n\}$ converges if and only if it is a Cauchy sequence.	8m
2	52	Discus the convergence of the sequence $\{x_n\}$ where $x_1 = 1,\ x_2 = 2,\ x_{n+2} = \frac{x_{n+1}+x_n}{2}$.	4m
2	53	Show that the sequence $\{x_n\}$ defined by $x_1 = \sqrt{2}$ and $x_{n+1} = \sqrt{2a_n}$ converges to 2.	4m
2	54	A sequence $\{x_n\}$ is defined as $x_1 = 1$ and $x_{n+1} = \frac{4+3x_n}{3+2x_n}, n \geq 1$. Show that $\{x_n\}$ converges and find its limit.	4m
2	55	Prove that: (i) If $p > 0$ then $\lim\limits_{n\to\infty} n^{\frac{1}{n}} = 1$. (ii) If $p > 0$ and $\alpha \in \mathbb{R}$ then $\lim\limits_{n\to\infty} \frac{n^\alpha}{(1+p)^n} = 0$.	6m
2	56	State and prove the Cauchy's first limit theorem.	6m
3	57	Discuss the convergence of $\sum\limits_{n=1}^{\infty} \frac{1}{n^p}$, $p \in \mathbb{R}$.	8m
3	58	Suppose $a_1 \geq a_2 \geq \ldots, \geq a_n \geq 0$ then show that $\sum\limits_{n=1}^{\infty} a_n$ converges if and only if $\sum\limits_{n=1}^{\infty} 2^k a_{2k}$ converges.	8m
3	59	State and prove the Cauchy's Criterion for convergence of a series.	6m
3	60	Define the number e. Show that it is the limit of the sequence $\{x_n\}$ where $x_n = \left(1+\frac{1}{n}\right)^n, n \in \mathbb{N}$.	6m
3	61	State and prove Integral test.	8m
3	62	Suppose $a_n > 0$ and $b_n > 0$ for $n \geq 1$ and $\lim\limits_{n\to\infty} \frac{a_n}{b_n} = \alpha$ where α is non zero real number. Show that $\sum\limits_{n=0}^{\infty} a_n$ and $\sum\limits_{n=0}^{\infty} b_n$ behave alike.	4m
3	63	State and prove integral test.	8m
3	64	State and prove Gauss test.	6m
3	65	Investigate the behavior of the series $\sum\limits_{n=1}^{\infty} \frac{(n!)^2}{(2n)!}\, x^n, \quad x > 0$.	4m
3	66	Investigate the behavior of the series $\sum\limits_{n=1}^{\infty} e^{-KH_n}$ where $H_n = 1 + \frac{1}{2} + \frac{1}{3} + \cdots, + \frac{1}{n}$.	6m

3	67	Test for convergence of the hyper geometric series $\sum_{n=1}^{\infty} 1+\frac{\alpha.\beta}{1.\gamma}x+\frac{\alpha(1+\alpha)\beta(1+\beta)}{1.2\ \gamma(1+\gamma)}x^2+\cdots$, for all positive values of $x;\alpha,\beta,\gamma$ being all positive.	6m
3	68	Test the convergence of the series $\sum_{n=1}^{\infty}\frac{\sqrt{n+1}-\sqrt{n}}{n}$.	4m
3	69	Test the convergence of the series $\sum_{n=1}^{\infty}\frac{n^3+1}{2^n+1}$.	4m
3	70	Test the convergence of the series $\sum_{n=1}^{\infty}\frac{n^3+1}{2^n+1}$.	4m
3	71	Test the convergence of the series $\sum_{n=1}^{\infty}\frac{1.6.11.\ldots,(5n-4)}{2.6.10.\ldots,(4n-2)}$.	4m
3	72	Test the convergence of the series $\sum_{n=1}^{\infty}\frac{1}{3^n-\sin n}$.	4m
3	73	Test the convergence and absolute convergence of the series $\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n\sqrt{n}}$.	4m
3	74	Test the convergence and absolute convergence of the series $\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n+1}$.	4m
4	75	State and prove Riemann's Rearrangement theorem.	10m
4	76	State and prove Cauchy condition for convergence of infinite product.	8m
4	77	Define Cauchy product of two series and show that Cauchy's product of two convergent series need not be convergent.	6m
4	78	State and prove Mertens theorem.	8m
4	79	Suppose that $a_n\geq 0,\ \forall n\in\mathbb{N}$, then show that $\prod_{n=1}^{\infty}(1+a_n)$ converges if and only if $\sum_{n=1}^{\infty}a_n$ converges.	6m
4	80	Suppose $\sum a_n$ is a rearrangement of $\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n}$, in which p positive terms are followed by q negative terms, in which $p\geq q\geq 1$. Show that $\sum a_n=\log 2+\frac{1}{2}\log\left(\frac{p}{q}\right)$.	6m
4	81	Show that $\prod_{n=2}^{\infty}\left(1+\frac{1}{2^n-2}\right)=2\sum_{n=1}^{\infty}2^{-n}$.	4m
4	82	Show that $\prod_{n=2}^{\infty}\left(1+\frac{1}{n^2-1}\right)=2\sum_{n=1}^{\infty}\frac{1}{n(n+1)}$.	4m
4	83	Show that $\prod_{n=2}^{\infty}\left(1+\frac{2n+1}{(n^2-1)(n+1)^2}\right)=\frac{4}{3}$	4m

4	84	Show that the Cauchy product of the convergent series $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n+1}}$ with itself is not convergent .	4m

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St. Philomena's College (Autonomous), Mysore

M. Sc-Mathematics (CBCS)

I/II/III/IV- Semester Examination: 2020-21

Subject:

Time: 3 Hours **Max Marks: 70**

Sl. No			Marks
Section – A (MCQ)			
1	a		1
	b		1
	c		1
	d		1
Section – B			
2	a		2
	b		2
	c		2
Section – C **Answer any three from the following**			
	3		3x10=30
	4		
	5		
	6		
Section – D **Answer any three from the following**			
	7		3x10=30
	8		
	9		
	10		

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