St.Philomena's College (Autonomus), Mysore
PG Department of Mathematics
Question Bank (Revised Curriculum 2020 onwards)
First Year - First Semester (2021 -23 Batch)
Course Title (Paper Title): Real Analysis -II

Unit	S.No	Question	Marks
1	1	Define countable and uncountable set	2m
1	2	Construct a bounded set of real numbers with exactly three limit points	2m
1	3	Is the collection of disjoint intervals of positive length countable set ? why?	2m
1	4	The E be a non empty connected subset of $\mathbb{R}$ and E contains no irrational numbers find the cardinality of E.	2m
1	5	Is the set of intervals with rational endpoints a countable set? Why?	2m
2	6	Show that the collection of all subsets of positive integers is uncountable	2m
2	7	Give an example of a real valued continuous function f such that set off all zeros of f is a closed set but not compact	2m
2	8	Give an example of a function f which is bounded but not of bounded variation	2m
3	9	Between any two roots of $\sin x$ show that there is a root of $\cos x$	2m
3	10	Draw the graph of $f(x) = [x]$ what kind of discontinuity do the function f has ?	2m
3	11	Show that there is no value ke such that the equation $x^3 - 3x + k = 0$ has two distinct roots in [0,1].	2m
3	12	Construct a function $f : \mathbb{R} \to \mathbb{R}$ which is continuous precisely at one point.	2m
3	13	Given open cover of the interval which is no finite sub cover	2m
4	14	Prove that $\forall x > 0$, $sinx > x - \frac{x^3}{6}$	2m
4	15	If $f(x)$ equals to zero $\forall x \in \mathbb{Q}$ and equals to 1 for all irrational x show that f is not Riemann integrable on [a,b] for any $a < b$.	2m

4	16	Give an example of a function whose derivative is not continuous	2m
4	17	Suppose $f \geq 0$, f is continous on [a,b] and $\int_a^b f(x)dx = 0$. Prove that $f(x) = 0 \;\; \forall x \in [a, b]$	2m
1	18	Prove that the open interval (0,1) is uncountable. In a metric space,prove that a finite intersection of open sets is open.	10m
1	19	If $\{I_n\}$ is a sequence of intervals $\in \mathbb{R}$ such that $I_n \supset I_{n+1}, n \in \mathbb{N}$ then show that $\cap I_n$ is not empty. Prove that a closed interval [a,b] is compact by showing that every open cover of [a,b] has a finite subcover.	10m
1	20	Prove that open interval (a,b) is an open set. Prove that every infinite subset of a countable set is countable.	10m
1	21	If $\{E_n\}$ is a sequence of countable set then show that $\cup_{n=1}^{\infty} E_n$ is countable. If A_1 and A_2 are countable show that $A_1 \times A_2$ is countable.	10m
1	22	Prove that every neighbourhood is an open set by defining a neighbourhood of a point .If x is a limit point of E, then prove that every neighbourhood of x contains infinitely many points of E	10m
1	23	Show that a set G is open if and only if G^C is closed. If X is a metric space and $E \subset X$.Then prove that E is closed if and only if $\bar{E} = E$	10m
2	24	Prove that every open set in $\mathbb{R}$ is the union of at most countable collection of disjoint open intervals .	10m
2	25	If X and Y are metric spaces, $E \subset X$ and p be a limit point of E, if $f : E \to Y$ then prove that $\lim_{x \to p} f(x) = q$ if and only if $\lim_{n \to \infty} f(x_n) = q$ for every sequence $\{x_n\}$ in E with $x_n \neq p$ .Prove that a mapping f of a metric space X into a metric space Y is continous on X if and only if $f^{-1}(G)$ is open in X for every open set G in Y.	10m
2	26	Prove that a continuous image of a compact set is compact. Prove that a subset of $\mathbb{R}$ is connected if and only if it is an interval	10m

2	27	Let A be a countable set and B_n be the set of all n-tuples $(a_1, a_2, a_3 \cdots a_n)$ where $a_i \in A, i = 1, 2, 3 \cdots n$. Prove that B_n is countable and hence deduce that the set of all rational numbers $\mathbb{Q}$ is countable . If (X, d) is a metric space . Define $d_1 : X \times X \to \mathbb{R}$ by $d_1(x, y) = \dfrac{d(x, y)}{1 + d(x, y)}$. Then prove that d_1 is a metric on X	10m
2	28	Prove that continuous image of a connected set is connected. Deduce intermediate value theorem	10m
3	29	Prove that monotonic functions have no discontinuity of second kind. Show that $\dfrac{x}{1+x} < log(1 + x) < x, \ \ \forall \ \ x > 0$	10m
3	30	State and prove generalised mean value theorem. Let f be a real valued function defined on [a,b] and suppose $f''(x) \geq 0 \ \forall x \in [a, b]$. Prove that $f(\frac{a+b}{2}) \leq \frac{1}{2}[f(a) + f(b)]$	10m
3	31	Prove that a real valued continous function defined on compact $E \subset \mathbb{R}$ is uniformly continous	10m
3	32	If f is monotonic on (a,b). Prove that set of points of (a,b) at which f is discontinuous is at most countable If $c_0 + \dfrac{c_1}{2} + \dfrac{c_2}{3} + \cdots + \dfrac{c_{n-1}}{n} + \dfrac{c_n}{n+1} = 0$, where $c_0, c_1, ..., c_n$ are real constants. Prove that the equation $c_0 + c_1x + c_2x^2 + \cdots + c_nx^n = 0$ has at least one root between 0 and 1 .	10m
3	33	If f is monotically increasing on (a,b) then show that both $f(x^+)$ and $f(x^-)$ exist at everey point x of (a,b).	10m
3	34	Construct a function f monotonic on (0,1); discontinous at every rational point of (0,1) and at no other point of (0,1). If f is a real continous function defined on a closed set $E \subset \mathbb{R}$. Prove that there is a continous extension of f from E to $\mathbb{R}$	10m
3	35	Let f be continous on [a,b] and f' exist and bounded on (a,b) then prove that f is of bounded variation on [a,b] .	10m

3	37	Let f be of bounded variation on [a,b]: then prove that f is of bounded variation on [a,c] and on [c,b] and that $V_a^b(f) = V_a^c(f) + V_c^b(f)$	10m
4	38	If P^* is a refinement of P then prove that $L(P, f, \alpha) \leq L(P^*, f, \alpha)$ and $U(P^*, f, \alpha) \leq U(P, f, \alpha)$. If f is continous on [a,b] then prove that $f \in R(\alpha)$ on [a,b]	10m
3	39	If $f, g \in R(\alpha)$ on [a,b] then prove that $f+g \in R(\alpha)$ and that $\int_a^b (f+g) d\alpha = \int_a^b f d\alpha + \int_a^b g d\alpha$.	10m
3	40	If $\gamma'$is continous on[a,b] then prove that γ is rectifable and $\Lambda(\gamma) = \int_a^b \lvert\gamma'(t)\rvert dt$	
2	41	Suppose α is monotonically increasing and α' is Riemann integrable on [a,b]. Let f be bounded real function on [a,b] then prove that $f \in R(\alpha)$ if and only if $f\alpha' \in R$ on [a,b]	10m
4	43	State and prove Taylors theorem	10m
4	44	Let $f \in R$ on [a,b] and f is continous at $x_0 \in [a, b]$ for $a \leq x \leq b$ put $F(x) = \int_a^b f(t) dt$. Prove thst F is continous on [a,b] and $F'(x_0) = f(x_0)$. Suppose f is a continous function from [0,1] to [0,1] prove that $f(x) = x$ for at least one $x \in [0, 1]$	10m

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St. Philomena's College (Autonomous), Mysore

M. Sc-Mathematics (CBCS)

I/II/III/IV- Semester Examination: 2020-21

Subject:

Time: 3 Hours Max Marks: 70

Sl. No			Marks
Section – A (MCQ)			
1	a		1
	b		1
	c		1
	d		1
Section – B			
2	a		2
	b		2
	c		2
Section – C Answer any three from the following			
	3		3x10=30
	4		
	5		
	6		
Section – D Answer any three from the following			
	7		3x10=30
	8		
	9		
	10		