St.Philomena's College (Autonomus), Mysore

PG Department of Mathematics

Question Bank (Revised Curriculum 2018 onwards)

First Year - First Semester (2020 -22 Batch)

Course Title (Paper Title): Introduction to Complex analysis

Q.P.Code-57202

Unit	Sl.No	Question	Marks
1	1	Prove that $\|z+w\| \geq \|z\| - \|w\|$.	2m
1	2	Prove that if z is a root of the polynomial equation $P(z) = 0$, then $\bar{z}$ is also a root.	2m
1	3	Prove that if $\|z\| = Re(z)$, then z is a non-negative real number.	2m
1	4	Let z be a complex number such that $Re(z) > 0$. Prove that $Re(\frac{1}{z}) > 0$.	2m
1	5	Prove that $\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\overline{z_1}}{\overline{z_2}}, \quad (z_2 \neq 0)$.	2m
2	6	Prove that the centroid of the triangle whose vertices are z_1, z_2, z_3 is $\frac{z_1+z_2+z_3}{3}$.	2m
2	7	Show that the function $w = 2\overline{z}^2 + 1$ is defined in the entire complex plane.	2m
2	8	Compute $\lim_{z \to (1+i)} \left\{ \frac{z-1-i}{z^2-2z+2} \right\}^2$.	2m
2	9	Find the value of the principle square root function $z^{\frac{1}{2}}$ at $z = -2i$.	2m
2	10	Give an example of a function which is continuous everywhere and no where analytic.	2m

3	11	If $f(z)$ and $f(\bar{z})$ are both analytic in a region D. Show that $f(z)$ is constant in the region.	2m
3	12	Prove that the limit of the sequence is unique if it exists.	2m
3	13	Verify Cauchy-Riemann equations for the function z^3.	2m
3	14	Show that $\lim\limits_{z \to 0} \frac{z}{\bar{z}}$ does not exists.	2m
4	15	Let $\gamma(t) = t^2 + it^2$ for $-1 \leq t \leq 1$. Discuss the properties of γ and also sketch a rough graph of the trajectory of γ.	2m
4	16	Evaluate $\oint_c \frac{1}{z}$, where c is the circle $x = cos\, t,\ y = sin\, t,\ 0 \leq t \leq 2\pi$.	2m
4	17	Evaluate $\int_c \frac{z^3-2z+1}{(z-i)^2}\, dz$, where c is the circle $\lvert z\rvert = 2$.	2m
1	18	State and prove Lagrange's identity in the complex form. Hence, deduce Cauchy's inequality.	10m
1	19	Given that $z \neq 0$ and $w \neq 0$, demonstrate that $\lvert z + w\rvert = \lvert z\rvert + \lvert w\rvert$ is true if and only if $w = tz$, for some $t > 0$. State and prove Demoivre's formula for complex number.	10m
1	20	If z =x+i y and $w^2 = z$ for some w. Then find the value of w in complex form. Determine all the solutions of the equation $z^4 + 16 = 0$.	10m
1	21	Write a note on Riemann Sphere and hence deduce the formula for the distance between two points on it.	10m

1	22	Prove that two points z_1 and z_2 are reflection points for the line $\overline{\alpha}z + \alpha\overline{z} + r = 0$ if and only if $\overline{\alpha}z_1 + \alpha\overline{z_2} + r = 0$. If z_1, z_2, z_3 are the vertices of an isosceles right angled at the vertices z_2, prove that $z_1^2 + 2z_2 + z_3^2 = 2z_2(z_1 + z_3)$.	10m
1	23	The points z_1, z_2, z_3, z_4 taken in order are con cyclic if and only if $\frac{(z_3 - z_1)(z_4 - z_2)}{(z_3 - z_2)(z_4 - z_1)}$ is purely real. Also, if $z_1z_2 + z_3z_4 = 0$ and $z_1 + z_2 = 0$, then z_1, z_2, z_3, z_4 are con cyclic. Show that two points z_1, z_2 will be inverse points with respect to the circle $z_1\overline{z_2} + \overline{\alpha}z_1 + \alpha\overline{z_2} + r = 0$.	10m
1	24	Derive the general equation of the straight line in complex plane. Show that the triangle whose vertices are the oints represented by the complex numbers z_1, z_2, z_3 on the argand diagram is equilateral if and only if $\frac{1}{z_2 - z_3} + \frac{1}{z_3 - z_1} + \frac{1}{z_1 - z_2} = 0.$	10m
1	25	Evaluate $\lim_{z \to (e^{i\frac{\pi}{3}})} (z - e^{i\frac{\pi}{3}}) \frac{z}{(z^3 + 1)}$. Define continuity of a function and check whether the function $f(z) = \frac{Re(z)^2}{\|z\|^2}$ is continuous at $z = 0$.	10m
2	26	Show that the function $f(z) = \overline{z}$ is continuous everywhere but no where differentiable. State and prove the necessary condition for a function to be analytic.	10m
2	27	Deduce the polar form of Cauchy-Riemann equation and show that $f(z) = \sqrt{r}(cos\frac{\theta}{2} + isin\frac{\theta}{2})$, where $r > 0$ and $0 < \theta < 2\pi$ is differentiable. If $f(z)$ is differentiable function, show that the Cauchy-Riemann equations can be put in the form $\frac{\partial f}{\partial \overline{z}}$.	10m

2	28	Show that if $f(z) = u + iv$ is analytic function, then u and v are both harmonic functions. Find the analytic function $f(z) = u + iv$ given that $u - v = e^x(cosy - siny)$.	10m
2	29	Define a sequence, convergent sequence and cauchy sequence. Prove that a sequence $\{z_n\}$ is convergent if and only if the two real sequence $\{Re(z_n)\}$ and $\{Im(z_n)\}$ are both convergent and also, $$\lim_{n\to\infty} z_n = \lim_{n\to\infty} \{Re(z_n) \ + i \lim_{n\to\infty} \{Im(z_n) \ .$$	10m
2	30	State and prove Cauchy general principle of convergence for a sequence.	10m
2	31	Write a short note about the absolute convergence of a series. Discuss some tests for convergence of infinite series.	10m
2	32	State and prove Abel's limit theorem. Show that for any two complex numbers z_1 and z_2, $e^{z_1+z_2} = e^{z_1} \cdot e^{z_2}$.	10m
3	33	Define a neighborhood of a point, open set, closed set, limit point of a set, exterior point of a set, connected set and a region. Show that under the transformation $iz + i$, the half plane $x > 0$ maps onto the half plane $v > 1$.	10m
3	34	Any bilinear transformation can be expressed as a product of transformation, rotation, magnification or contraction and inversion. Find the image of the circle $\|z - 3i\| = 3$ under the map $w = \frac{1}{z}$.	10m

3	35	Define Cross Ratio and thus, show that any bilinear transformation preserves cross ratio. If c is a curve $y = x^3 - 3x^2 + 4x - 1$ joining points (1,1) and (2,3). Find the value of $\int_c (12z^2 - 4iz)\, dz$.	10m
2	36	Find the bilinear transformation which maps the points $z = -1, 1, \infty$ respectively on $w = -i, -1, i$. Evaluate $\int\limits_{(0,3)}^{(2,3)} (2y + x^2)dx + (3x - y)dy$ along: 1. the parabola $x = 2t,\ y = t^2 + 3$. 2. straight lines from (0,3) to (2,3) and then from (2,3) to (2,4). 3. a straight line from (0,3) to (2,4)	10m
3	37	Integrate the functions $f(z) = x + y - 3ix^3$ and $g(z) = -x + (y + z)i$ between the points $z = 0$ and $z = 1+i$ along different oaths. Let $w(t)$ be a complex valued function integrable on [a,b]. Then $\left\| \int\limits_a^b w(t)\, dt \right\| \leq \int\limits_a^b \|w(t)\|$.	10m
3	38	State and prove ML-inequality and hence find an upper bound for the absolute value of $\oint_c \frac{e^z}{z+1} dz$ where c is the circle $\|z\| = 4$. Evaluate $\int_c \overline{z} dz$, where$c$ is given by $x = 3t,\ y = t^2,\ -1 \leq t \leq 4$.	10m
4	39	State and prove Cauchy's theorem for a triangle.	10m
4	40	If $f(z)$ is analytic and z_1, z_2 are any two points in a single connected region R, then show that $\int\limits_{z_1}^{z_2} f(z)dz$ is independent of the path in R joining z_1 and z_2.Evaluate $\int_c \frac{dz}{z-a}$, where c is any simple closed curve, when $z = a$ is outside c and $z = a$ is inside c.	10m

4	41	State and prove Cauchy's Integral formula. Evaluate $\int\limits_{\lvert z\rvert=1} \frac{cos\, 2\pi z}{(2z-1)(z-3)}$.	10m
4	42	State and prove Morera's theorem. Evaluate $\int_c \frac{dz}{(z-i)(z+i)}$, where c is circle $\lvert z+1\rvert = 1$.	10m
4	43	State and prove Cauchy's Inequality. If $F(a) = \int_c \frac{z^2+2z-1}{(z-a)^2} dz$, where c is the ellipse $\dfrac{x^2}{4} + \dfrac{y^2}{9} = 1$, find $F'(1)$, $F'(i)$, $F'(-i)$.	10m
4	44	State and prove Liouville's theorem, and deduce fundamental theorem of algebra from it.	10m
4	45	State and prove Maximum modulus theorem. If $f(z) = z^2 + 2$, then determine the minimum value of $\lvert f(z)\rvert$ over the closed region $\lvert z\rvert \leq 1$.	10m

Blue Print of the Question Paper

St. Philomena's College (Autonomous), Mysore

M. Sc-Mathematics (CBCS)

I/II/III/IV- Semester Examination: 2020-21

Subject:

Time: 3 Hours **Max Marks: 70**

Sl. No			Marks
		Section – A (MCQ)	
1	a		1
	b		1
	c		1
	d		1
		Section – B	
2	a		2
	b		2
	c		2
		Section – C Answer any three from the following	
	3		3x10=30
	4		
	5		
	6		
		Section – D Answer any three from the following	
	7		3x10=30
	8		
	9		
	10		