St.Philomena's College (Autonomus), Mysore

PG Department of Mathematics

Question Bank (Revised Curriculum 2020 onwards)

First Year - First Semester (2020 -22 Batch)

Course Title (Paper Title): Linear Algebra with Applications Q.P.Code-

Unit	S.No	Question	Marks
1	1	Define vector space.	2m
1	2	Define subspace of a vector space.	2m
1	3	Prove that intersection of any two subspaces is also a subspace.	2m
1	4	Is union of any two subspaces is also a subspace? Justify.	2m
1	5	Define linear combination of vectors.	2m
1	6	Define linear span of a set.	2m
1	7	Define linearly independent set.	2m
1	8	Define linearly dependent set.	2m
1	9	Show that $W = \{(x, y, z)/lx + my + nz = 0\}$ is a subspace of $\mathbb{R}^3$.	2m
1	10	Prove that if two vectors are linearly dependent one of them is a scalar multiple of the other.	2m
1	11	If $v_1, v_2, v_3 \in V(F)$ such that $v_1 + v_2 + v_3 = 0$ then show that $\{v_1, v_2, \}$ spans the same subspace of $\{v_2, v_3\}$.	2m

1	12	Define basis of a vector space.	2m
1	13	Prove that any two basis of a finite dimensional vector space V have the same number of elements.	2m
1	14	Define maximal linearly independent set.	2m
1	15	Define minimal generating set.	2m
1	16	Define Kernel of a linear transformation.	2m
1	17	Define dimension of a vector space.	2m
1	18	Define linear transformation.	2m
1	19	Define range space.	2m
1	20	Define dual space of a vector space.	2m
2	21	Define inner product space.	2m
2	22	Define norm and find the norm of (4,5,6).	2m
2	23	Define an orthogonal complement. Find the orthogonal compliment of y-axis.	2m
2	24	If V be a vector space of polynomials with inner product given by $(f,g) = \int_0^1 f(t)g(t)dt$ then find (f,g) if $f(t) = t+2$ and $g(t) = t^2 - 2t - 3$.	2m
2	25	If V be a vector space of polynomials with inner product given by $(f,g) = \int_0^1 f(t)g(t)dt$ then find $\|f\|$ if $f(t) = t^3 - 3$.	2m
2	26	In any inner product space show that $\|\alpha u\| = \|\alpha\|\|u\| \ \forall \alpha \in F \ u \in V$.	2m

2	27	Prove that absolute value of cosine of angle is almost one.	2m
2	28	Define orthogonal set and orthonormal set.	2m
2	29	Obtain an orthonormal basis with respect to the standard inner product for the subspace of R^3 is generated by $(1,0,3)$ and $(2,1,1)$.	2m
2	30	If V is a finite dimensional inner product space and W is a subspace of V then prove that $(W^{\perp})^{\perp} = W$.	2m
2	31	Solve the following system of equations by cramer's rule $x_1+2x_2+3x_3 = -5$, $2x_1+x_2+x_3 = -7$, $x_1+x_2+x_3 = 0$.	2m
2	32	Consider the inner product space R^4 with the standard inner product. If $u = (3,2,k,-5)$ and $v = (1,k,7,3)$ are orthogonal. Find the value of k.	2m
2	33	If u and v are vectors in an inner product space V, then prove that $\|u+v\|^2 + \|u-v\|^2 = 2\left(\|u\|^2 + \|v\|^2\right)$	2m
2	34	Define bilinear form.	2m
2	35	Define symmetric and skew-symmetric bilinear form.	2m
3	36	Define eigen value and eigen vector of T.	2m
3	37	Define normal linear operator.	2m
3	38	Define Hermitian adjoint of T and prove that $(T^*)^* = T$.	2m
3	39	Define unitary transformation.	2m
3	40	If $T \in A(V)$ is unitary then prove that $TT^* = I = T^*T$.	2m

3	41	Prove that T is unitary then $\lVert Tv \rVert = \lVert v \rVert$.	2m
3	42	If T is unitary and λ is a characteristic root of T, then prove that $\lvert\lambda\rvert = 1$.	2m
3	43	Show that $A = \begin{pmatrix} 4 & 3-5i \\ 3+5i & -7 \end{pmatrix}$ is hermitian.	2m
3	44	Show that $A = \begin{pmatrix} 4i & 3+2i \\ -3+2i & -7i \end{pmatrix}$ is skew hermitian.	2m
3	45	Show that $A = \begin{pmatrix} 2+3i & 1 \\ i & 1+2i \end{pmatrix}$ is normal.	2m
3	46	Show that $A = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$ where $\theta$is real, is unitary.	2m
4	47	Define a minimal polynomial for T over F.	2m
4	48	Determine the rank and signature of the matrix $A = \begin{pmatrix} 4 & 5 & -2 \\ 5 & 1 & -2 \\ -2 & -2 & 3 \end{pmatrix}$.	2m
4	49	Find the minimal polynomial of the matrix $\begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & -9 \\ 0 & 1 & 6 \end{pmatrix}$.	2m

4	50	Reduce to Jordan form the matrix $\begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \end{pmatrix}$	2m
4	51	Find the symmetric matrix A associated with $q(x,y) = 2x^2 + xy + y^2$	2m
4	52	Find the rank and signature of the quadratic form $x_1^2 - 4x_1x_2 + x_2^2$.	2m
4	53	Reduce to triangular form, the matrix $A = \begin{pmatrix} 1 & 1 & 1 \\ -1 & -1 & -1 \\ 1 & 1 & 0 \end{pmatrix}$	2m
4	54	Define companion matrix of a function.	2m
4	55	Give an example of two matrices which have the same minimum polynomial but are not similar to each other.	2m
4	56	If $\lambda \in F$ is a characteristic root of $T \in A(V)$ and if $q(x) \in F[x]$ is a polynomial then prove that $q(\lambda)$ is a characteristic root of $q(T)$.	2m
4	57	If A is a lower triangular matrix then prove that all the diagonal entries are eigen values of A.	2m
1	58	Show that sum of any two subspaces is also a subspace.	3m
3	59	Let $T \in A(V)$ be a Skew - Hermitian transformation then prove that all the eigen values of T are purely imaginary.	4m

3	60	If $T \in A(V)$ and if $(Tv, Tv) = (v, v)$ for all $v \in V$ then prove that T is unitary.	4m
3	61	Let T be a self adjoint operator on a finite dimensional inner product space V. Then prove that every eigen values of T are real.	4m
3	62	If λ is a characteristic root of the normal transformation N and if $vN = \lambda v$ then prove that $vN^* = \overline{\lambda} v$.	4m
3	63	If $T \in A(V)$ is such that $(vT, v) = 0$ for all $v \in V$, then prove that $T = 0$.	4m
4	64	Find the rank and signature of the quadratic form $x_1^2 + 2x_1x_2 + x_2^2$.	4m
1	65	Show that the set of complex numbers is a vector space over the field of real numbers.	5m
3	66	If $v_1, v_2, \ldots, v_n$ is an orthonormal basis of V and if the matrix of $T \in A(V)$ in this basis is (α_{ij}), then prove that the matrix of T^* in this basis is (β_{ij}), where $(\beta_{ij}) = \overline{(\alpha_{ij})}$.	5m
3	67	Prove that the Hermitian adjoint is a linear transformation.	5m
3	68	Prove that the linear transformation T on V is unitary if and only if it takes an orthonarmal basis of V into an orthonormal basis of V.	5m
3	69	Let $T \in A(V)$ be a unitary transformation and let λ be an eigen value of T then show that $|\lambda| = 1$.	5m

1	70	Prove that the necessary and sufficient condition for a non-empty subset W of a vector space V over a field F to be a subspace of V is that W is closed under vector addition and scalar multiplication.	6m
1	71	Let V be a finite dimensional vector space over a field F then prove that any linearly independent set of vectors in V is a part of a basis	6m
1	72	If the system of homogeneous linear equation $a_{11}x_1+a_{12}x_2+\ldots+a_{1n}x_n = 0$, $a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n = 0, \ldots, a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n = 0$, where $a_{ij} \in F$is of rank r then prove that there are n-r linear independent solutions in $F^{(n)}$.	6m
2	73	Show that any orthonormal set is linearly independent.	6m
2	74	If $u, v \in V$ and $\alpha, \beta \in F$ then show that $(\alpha u+\beta v, \alpha u+\beta v) = \alpha\overline{\alpha}(u, u) + \alpha\overline{\beta}(u, v) + \beta\overline{\alpha}(v, u) + \beta\overline{\beta}(v, v)$.	6m
2	75	Let V be the set of all continuous real valued function defined on the closed interval $[0, 1]$ Show that V is a real inner product space with the inner product defined by $(f, g) = \int_0^1 f(t)g(t)dt$.	6m
3	76	Prove that the element $\lambda \in F$ is the characteristic root of $T \in A(V)$ if and only if for some $v \neq 0$ in V, $vT = \lambda v$.	6m
3	77	Suppose S and T are linear operator on an inner product space V and c is a scalar. If S and T possess adjoint operator. Prove that $S + T$, cT, ST possess adjoint.	6m

4	78	If $T, S \in A(V)$ and if S is regular, then T and STS^{-1}have the same minimal polynomial.	6m
4	79	Define the Jordan canonical form with suitable example which contains at least two Jordan blocks. Find the Jordan canonical form for the matrix: $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -2 \\ 0 & 1 & 3 \end{pmatrix}$	6m
1	80	Let V be a vector space over a field F then Prove that $S = \{v_1, v_2, \ldots, v_n\}$ is a basis of V if and only if every element of V can be uniquely expressed as a linear combination of finite element of S.	7m
1	81	State and prove Fundamental theorem of homomorphism.	7m
1	82	If $v_1, v_2, \ldots, v_n$ are in V then prove that either they are linearly independent or some v_k, $2 \leq k \leq n$ is a linear combination of the preceding vectors $v_1, v_2, \ldots, v_{(k-1)}$.	7m
1	83	Let V be a vector space over a field F. Let $S = \{v_1, v_2, \ldots, v_n\}$ and L(S)=W then Prove that if their exist a linearly independents of set S' of S such that $L(S') = W$.	7m
1	84	Let V be a vector space over a field F. Let $S = \{v_1, v_2, \ldots, v_n\}$ spans V and $S' = \{w_1, w_2, \ldots, w_m\}$ be a linearly independent set of vectors in V then show that $m \leq n$.	7m

1	85	Show that any vector space of dimension n over a field F is isomorphic to $V_n(F)$.	7m
2	86	Let V be a finite dimensional inner product space. Let W be a subspace of V. Prove that V is direct sum of W and $W^{\perp}$ that is $V = W \oplus W^{\perp}$.	7m
2	87	If V is a finite dimensional inner product space and W is a subspace of V then prove that $(W^{\perp})^{\perp}$.	7m
2	88	Let V be the set of real valued function $y = f(x)$ satisfying $\frac{d^2y}{dx^2} + 4y = 0$. Prove that V is a two dimensional real vector space. Define $(u, v) = \int_0^{\Pi} uvdx$, Find an orthonormal basis in V. (7M)	7m
2	89	Show that $V_n(\mathbb{C})$ is a complex inner product space with the inner product defined by $(u, v) = x_1\overline{y_1} + x_2\overline{y_2} + \cdots + x_n\overline{y_n}$ where $u = (x_1, x_2, \ldots, x_n)$ and $v = (y_1, y_2, \ldots, y_n)$.	7m
4	90	Prove that congruence is an equivalence relation.	7m
4	91	If V is finite dimensional vector space then $T \in A(V)$ is singular if and only if their exist $v \neq 0$ in V such that vT=0.	7m
4	92	If V is finite dimensional over F, then prove that $T \in A(V)$ is invertible if and only if the constant term of the minimal polynomial for T is not zero.	7m

4	93	If V is n-dimensional vector space over F and if $T \in A(V)$ has all its characteristic roots in F then prove that T satisfies a polynomial of degree n over F.	7m
1	94	If W is any subspace of a vector space V over a field F then prove that the set V/W of all cosets $W + v$, where $v \in V$ is a vector space over a field F for the vector addition and scalar multiplication defined as follows i) $W + v_1 + W + v_2 = W + v_1 + v_2 \ \forall \ v_1, v_2 \in V$ ii) $\alpha(W + v) = W + \alpha v$ $\forall \ \alpha \in F, v \in V$.	8m
1	95	Let $\{v_1, v_2, \ldots, v_n\}$ be basis of V define $\hat{v_i}(\alpha_1 v_1 + \alpha_2 v_2 + \ldots + \alpha_n v_n) = \alpha_i \ \forall i = 1, 2, \ldots, n$ then prove that $\hat{v_i}$ is a linear transformation $\forall i = 1, 2, \ldots, n$ and $\{\hat{v_1}, \hat{v_2}, \ldots, \hat{v_n}\}$ form a basis of $\hat{v}$ and hence dim V = dim $\hat{v}$.	8m
2	96	State and prove the Gram - Schmidt orthogonalization process.	8m
2	97	State and prove the Bessal's inequality.	8m
2	98	State and prove Cauchy - Schwartz Inequality.	8m
3	99	If $\lambda_1, \lambda_2, \ldots, \lambda_n \in F$ are distinct characteristic roots of $T \in A(V)$ and $v_1, v_2, \ldots, v_k$ characteristic vectors of T belonging $\lambda_1, \lambda_2, \ldots, \lambda_k$ respectively then prove that $v_1, v_2, \ldots, v_k$ linearly independent over F.	8m

4	100	If T in $A_F(V)$ has a minimal polynomial $p(x) = q(x)^e$, where $q(x)$, is a monic, irreducible polynomial in $F[x]$ then basis of V over F can be found in which the matrix of T is of the form $\begin{pmatrix} c(q(x)^{e_1}) & & & \\ & c(q(x)^{e_2}) & & \\ & & \ddots & \\ & & & c(q(x)^{e_r}) \end{pmatrix}$ where $e = e_1 \geq e_2 \geq \cdots \geq e_r$.(	8m
4	101	If W is a subset of V is invariant under T then T induces a linear transformation $\overline{T}$ on V/W defined by $\overline{T}(v + W) = T(v) + W$, if T satisfies the polynomial $q(x) \in F(x)$ then so does $\overline{T}$ and if $p_1(x)$ is the minimal polynomial for $\overline{T}$ over a field F and if p(x) for T then prove that $p_1(x)/p(x)$.	8m
3	102	Let V be an inner product space and let N be a normal transformation on V. Then prove that the following statements are true: i). $\|N(v)\| = \|N^*(v)\|\ \forall v \in V$ ii). $(N - cI)$ is normal, for every $c \in F$ iii). If λ_1 and λ_2 are distinct eigenvector of N with corresponding eigen values v_1 and v_2 then v_1 and v_2 are orthogonal.	10m
4	103	State and prove Sylvester's law of inertia.	10m

Blue Print of the Question Paper

St. Philomena's College (Autonomous), Mysore

M. Sc-Mathematics (CBCS)

I/II/III/IV- Semester Examination: 2020-21

Subject:

Time: 3 Hours **Max Marks: 70**

Sl. No			Marks
Section – A (MCQ)			
1	a		1
	b		1
	c		1
	d		1
Section – B			
2	a		2
	b		2
	c		2
Section – C **Answer any three from the following**			
	3		3x10=30
	4		
	5		
	6		
Section – D **Answer any three from the following**			
	7		3x10=30
	8		
	9		
	10		