St. Philomena's College (Autonomous), Mysore
Question Bank
Programme: M. Sc. Physics
I Semester
Course Title: Elements of Mathematical Physics
Course Type: Hard Core
Q.P Code:

Sl No.	Questions	Marks
1.	Discuss the nature of regular singular points and singulatities for a linear homogeneous second order differential equation with an example.	4
2.	Prove that $e^{\frac{x}{2}\left(t-\frac{1}{t}\right)}=\sum J_n(x)t^n$.	4
3.	Show that $x=0,\infty$ are respectively regular and irregular singular pointsand any other point is an ordinary point for Bessel equation	4
4.	Prove that $J_{n+1}(x)=\frac{n}{x}J_n(x)-J_n'(x)$.	4
5.	Prove that $\frac{d}{dx}\left(x^{-n}J_n(x)\right)=(-1)^n x^{-n}J_{n+1}(x)$.	4
6.	Prove that $\frac{d}{dx}\left(x^{n}J_n(x)\right)=x^n J_{n-1}(x)$.	4
7.	Prove that $J_{\frac{1}{2}}(x)=\sqrt{\frac{2}{\pi x}}\sin(x)$.	4
8.	Prove that $J_{\frac{-1}{2}}(x)=\sqrt{\frac{2}{\pi x}}\cos(x)$.	4
9.	Prove that $\left(1-2xt+t^2\right)^{\frac{-1}{2}}=\sum P_n(x)t^n$.	4
10.	Evaluate P_0 (x), P_1 (x) and P_2 (x) using Rodrigues representation.	4
11.	Prove that $P_{n+1}'(x)+P_{n-1}'(x)=2xP_n'(x)+P_n(x)$.	4
12.	Show that $(2n+1)P_n=(n+1)2xP_{n+1}+nP_{n-1}(x)$	4
13.	Calculate H_1 (x) and H_3 (x)using the Rodrigue's formula for Hermite polynomials.	4
14.	Calculate H_2 (x) and H_4 (x) using the Rodrigue's formula for Hermite polynomials.	4
15.	Prove that $\exp\left(-t^2+2xt\right)=\sum H_n(x)\frac{t^n}{n!}$.	4
16.	Show that $H_{n+1}(x)=2xH\,n(x)-2nH_{n-1}(x)$	4
17.	Show that $H_n(x)'(x)=2nH_{n-1}(x)$	4

18. Prove that $\frac{e^{-xz/(1-z)}}{(1-z)}=\sum L_n(x)Z^n$. 4

19. Show that $(n+1)L_{n+1}(x)=(2n+1-x)L_n(x)-nL_{n-1}(x)$ 4

20. Show that $xL_n'(x)=nL_n(x)-nL_{n-1}(x)$ 4

21. Obtain a series soluton for the Besses's differential equation 8

22. Obtain a series soluton for the Legendre differential equation at x=0. 8

23. Separate the Helmholtz equation using cartesian coordinates. 8

24. Separate the Helmholtz equation using spherical coordinates. 8

25. Separate the Helmholtz equation using circular cylindrical coordinates. 8

26. Derive the Orthogonality relations for the Hermite Polynomials 8

27. Obtain a series soluton to the Legendre differential equation. 8

28. Sketch and expand in a Fourier sine-cosine series the function $f(x)$given by $f(x)=\{0,-\pi<x<0;1,0<x<\pi$, using the Fourier series of $f(x)$write the Fourier series expansion of the function $g(x)$given by $g(x)=\{-1,-\%p<x<0;1,0<x<\pi$. 8

29. Sketch and expand in a Fourier sine-cosine series the function $f(x)$given by $f(x)=\{0,0<x<l;1,l<x<2l$. 8

30. Expand sinusoidal voltage of half wave rectifier in Fourier series. 8

31. Discuss even and odd functions and deduce coefficients of Fourier sine and cosine series. 8

32. Discuss Parseval's theorem and its inference in the context of sound. 8

33. Find the Fourier integral representation of $f(x)=\{1,|x|<1;0,|x|>1$. 8

34. Represent a non-periodic function $f(x)=\{1,|x|<1;0,|x|>1$ as a Fourier integral. 8

35. Deduce the integral equation for mass-spring system. 8

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38. Discuss conversion of Volterra equation to ODE. 8

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40. Discuss simple harmonic and wave motion and their relation to period functions. 9

41. Write short notes on applications of Fourier series. 9

42. Discuss average value of a function and obtain the average value of $\sin^2 nx$ and $\cos^2 nx$for the period 2π. 9

43. Deduce the Fourier coefficients in Fourier series of sines and cosines, for the period 2π . 9

44. Deduce the Fourier coefficients of Fourier series in complex form, for the period 2π. 9

45. Sketch and expand in a Fourier series in complex form the function $f(x)=\{0, -\pi<x<0; 1, 0<x<\pi$ and express the in sine-cosine series. 9

46. Discuss the conversion of following Boundary Value Problem to Fredholm integral equations, given by $y''(x)+P(x)y'(x)+Q(x)y(x)=f(x)$ with boundary conditions $x=a; y(a)=\alpha$ and $y=b; y(b)=\beta$. 9

47. Discuss the conversion of following Boundary Value Problem to Fredholm integral equations, given by $y''(x)=f(x, y(x)), 0\leqslant x\leqslant 1; y(0)=y_0; y(1)=y_1$. 9

48. Discuss the conversion of following Boundary Value Problem to Fredholm integral equations, given by $y''(x)+y(x)=x; 0<x<\pi/2; y(0)=1; y(\pi/2)=\pi$. 9

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52. Discuss the method of successive approximations to solve Volterra equations. 9

53. Discuss the method of Laplace transform to solve Volterra equations. 9

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55. Given $u(x)=f(x)+\lambda\int_0^x e^{-x-t}u(t)dt$, solve using method of successive approximations. 9

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61.	Discuss the method of Laplace transform to solve Volterra equations.	9
62.	Given $u(x)=f(x)+\lambda\int_0^x e^{-x-t}u(t)dt$, solve using method of method of successive approximations.	9
63.	Discuss active and passive transformations of a vector in 2D Cartesian coordinates.	9
64.	Discuss parallel and perpendicular projections in Cartesian and non-orthogonal coordinates.	9
65.	Discuss active and passive transformations of a vector in 2D Cartesian coordinates.	9
66.	Explain the concept of dual basis.	9
67.	Discuss the prototype of covariant tensor.	9
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72.	Discuss the prototype of contravariant tensor.	9
73.	Define contravariant and covariant tensor of rank 1 and by observation write the contravariant, covariant and mixed tensors of rank 2.	9
74.	Discuss addition and subtraction of contravariant and covariant tensors of rank 2.	9
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76.	Illustrate outer product and contraction of tensors.	9
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79.	Illustrate outer product and contraction of tensors.	9
80.	Justify the terminology “metric tensor”.	9
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91.	Obtain the expression for gradient using metric tensors and scale factors in a orthogonal coordinate system.	9
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99.	Obtain the expression for gradient using metric tensors and scale factors in a orthogonal coordinate system.	9
100.	Discuss Christoffel’s symbols.	9
101.	Explain covariant differentian.	9
102.	Derive the Orthogonality relations for the Legendre Polynomials	10
103.	Prove the orthogonality relations for Bessel functions	10
104.	Obtain a series soluton in even powers of x for the Hermite differential equation at x=0.	10
105.	Derive the Orthogonality relations for the Legendre Polynomials	10
106.	Discuss the orthogonality of trigonometric functions and deduce the Fourier coefficients in Fourier series of sines and cosines.	12
107.	Deduce the Fourier coefficients in Fourier series of sines and cosines, for the period 2π.	12

108. Deduce the Fourier coefficients in Fourier series of sines and cosines, for the period 2π. 12

109. Represent $f(x)=\{1, 0<x<1/2; 0, 1/2<x<1$ in Fourier series, Fourier sine series and Fourier cosine series. 12

110. Discuss application of Fourier series to sound. 12

111. Deduce Fourier integral from Fourier series. 12

112. Discuss the historical background of the Integral equations. 12

113. Discuss the tautochrone problem. 12

114. Discuss the classification of integral equations. 12

115. Discuss conversion of initial value problem to Volterra equations. 12

116. Discuss the classification of integral equations. 12

117. Discuss projections in non-orthogonal coordinates and justify the necessity of dual basis. 12