St.Philomena's College (Autonomus), Mysore

PG Department of Mathematics

Question Bank (Revised Curriculum 2020 onwards)

Second Year - Third Semester

Course Title (Paper Title):Elements Of Functional Analysis

Q.P.Code- 87321

Unit	S.No	Question	Marks
1	1	Define a complete metric space with an example.	2m
1	2	Show that $\mathbb{R}^n$ and $\mathbb{C}^n$ are complete metric spaces.	2m
1	3	Define a normed linear space	2m
1	4	Prove that every normed linear space is metric space	2m
1	5	Define a contraction map and a fixed point	2m
1	6	Define first category and second category metric spaces	2m
2	7	Show that ${l_n}^p(\mathbb{R})$ is a complete metric space	2m
2	8	Define a separable space with an example	2m
2	9	Find the dense subspace of $l^p(\mathbb{R})$	2m
2	10	Show that every dense subspace of $l^\infty(\mathbb{R})$ is uncountably infinite	2m
3	11	Is Bolzano Weirstrass property is true in general metric space? Justify	2m
3	12	Is Cantor's intersection property is true i general metric space? Justify	2m
3	13	Show that the boundedness in $\mathbb{R}$ implies total boundedness	2m

3	14	Define total boundedness of a set with an example	2m
3	15	Define linear operator and bounded linear operator	2m
4	16	Define a poset with an example	2m
4	17	Define Banach space with n example	2m
4	18	Define an open map	2m
4	19	Show that $l_2^\infty(\mathbb{R})$ is an Hilbert space	2m
4	20	In a Hilbert space define Orthogonality and Orthogonal Compliment	2m
1	21	Define complete metric space.Give two examples	4m
1	22	Define isometry with an example. Is every isometry is continuous? Justify .	4m
1	23	Define Nowhere Dense set and Everywhere dense subset with an example?	4m
2	24	Define first category and second category metric spaces.	4m
4	25	Prove that $\|\|x+y\|\|^2 - \|\|x-y\|\|^2 + i\|\|x+iy\|\|^2 - \|\|x-iy\|\|^2 = 4\|\|xy\|\|$	4m
4	26	In a Hilbert space show that inner product space is continuous.	6m
1	27	Show that every closed subspace of a complete metric space is complete.	7m
1	28	Define a normed linear space .Show that every normed linear space is a complete metric space	7m
2	29	Prove that a Normed linear space is complete if and only if every absolutely convergent series in X is convergent	7m

3	30	Show that the linear space $B(N, N')$ over $\mathbb{F}$ is a normed linear space with respect to $\|\|T\|\| = Sup\{ \ \|\|T(x)\|\|; \|\|x\|\| \leq 1 \ \}$	7m
4	31	In a Hilbert space show that inner product is continous.	7m
4	32	If M is a closed linear subspace of a Hilbert Space H then prove that $H = M \oplus M^{\perp}$	7m
4	33	In a Hilbert Space H , Define orthonormal set and prove Bessel's inequality.	7m
3	34	Define a an inner product space and Normed linear space . If $(H, (.,.))$ is an inner product space then prove that it is a Normed linear space	8m
1	35	Show that $B^*((x), d))$ is a complete metric space	10m
1	37	State and prove Metric completion theorem	10m
1	38	State and prove Banach fixed point theorem	10m
1	39	State and prove Baire's Category theorem.	10m
4	40	In a Hilbert space prove the Paralleogram law and Pythogoras theorem	10m
1	41	Prove Picards theorem as a consequence of Banach Fixed Point theorem	10m
2	42	Show that the following is equialent for the metric space (X, d) a) X is Compact b) X is Sequentially Compact c) X is complete and totally bounded	10m

2	43	If $T : N \to N'$ is a linear operator from an normed linear space N to a normed linear space N' then show that the following are equivalent a) T is continuous linear operator. b) T is continuous at $x = 0$. c) T is bounded linear operator. d) If $S = \{x \in N; \|\|x\|\| \leq 1\}$ is the closed unit sphere in N then $T(S)$ is bounded in N'	10m
3	44	State and prove Hahn Banach theorem for any Real linear space .	10m
3	45	State and prove Hahn Banach theorem for any Complex linear space .	10m
3	46	State and prove Open mapping theorem by explaining the definitions that are needed to prove the theorem.	10m
3	47	State and prove Closed Graph theorem by explaining the definitions that are needed to prove the theorem.	10m
3	48	State and prove Principle of uniform boundedness by explaining the definitions that are needed to prove the result.	10m

4	49	Let H be a Hilbert space such that $S \subset H$ then prove the following results a) $S \cap S^{\perp} = \{0\}$ b) $S^{\perp}$ is a closed subspace of H. c) $S_1 \subseteq S_2$ that implies $S_2^{\perp} \subseteq S_1^{\perp}$. d) $S \subseteq S^{\perp\perp}$	10m
4	50	If e_i is an orthonormal set in a Hilbert Space H then prove that the following is equivalent. a) e_i is complete. b) $x \perp e_i \implies x = 0$ c) $x \in H \implies x = \sum (x, e_i) e_i$. d) $x \in H \implies ||x||^2 = \sum ||(x, e_i)||^2$	10m

Model Question Paper

St.Philomena's College (Autonomous), Mysuru
M.Sc Mathematics
Third Semester Examination 2020-21
Subject - Elements of Functional Analysis

Time - 3 Hours | Maximum Marks-70

Section A

Answer the following questions **4 × 1= 4**

1. (a) Which of the following is not a complete metric space.

    i) $\mathbb{R}^n$ ii) $(\mathbb{R})$ iii) $[0,1]$ iv) $[0,1]-\{0\}$

   (b) The set of all limit point of the set $S=\left\{\frac{1}{m}+\frac{1}{n}:m,n\in\mathbb{N}\right\}$ is

    i) $\emptyset$ ii) $\{0\}$ iii) $\left\{\frac{1}{m};m\in\mathbb{N}\right\}$ iv) None of the above

   (c) A complete normed linear space is

    i) Banach space ii) Hilbert Space

    iii) Both Banach and Hilbert iv) None of the above

   (d) Orthonormal set $S^{\perp}$ of a Complete Metric Space is

    i) Banach space ii) Hilbert Space

    iii) Complete iv) Incomplete

Section B

Answer the following questions **3 × 2= 6**

2. (a) Define a contraction map and a fixed point.

   (b) Show that every dense subspace of $l^{\infty}(\mathbb{R})$ is uncountably infinite.

   (c) Is Bolzano Weirstrass property is true in general metric space? Justify.

Section C

Answer any *three* of the following questions **3 × 10= 30**

3. Show that every closed subspace of a complete metric space is complete.Also show that every normed linear space is a complete metric space.

4. State and prove Banach fixed point theorem .

5. State and prove Baire's Category theorem.

6. State and Prove Stone Weirstrass theorem.

Section D

Answer any *three* of the following questions **3 × 10= 30**

7. State and prove Hahn Banach theorem for real case.

8. State and Prove Closed graph theorem .

9. Let H be a Hilbert space such that $S \subset H$ then prove the following results

a) $S \cap S^{\perp} = \{0\}$

b) $S^{\perp}$ is a closed subspace of H.

c) $S_1 \subseteq S_2$ that implies $S_2^{\perp} \subseteq S_1^{\perp}$.

d) $S \subseteq S^{\perp\perp}$

10. If e_i is an orthonormal set in a Hilbert Space H then prove that the following is equivalent.

a) e_i is complete.

b) $x \perp e_i \implies x = 0$

c) $x \in H \implies \quad x = \sum (x, e_i) e_i$.

d) $x \in H \implies \quad ||x||^2 = \sum ||(x, e_i)||^2$

* * * * * * *