St.Philomena's College (Autonomus), Mysore

PG Department of Mathematics

Question Bank (Revised Curriculum 2020 onwards)

First Year - Third Semester

Course Title (Paper Title): Topology-I **Q.P.Code-87322**

Unit	Sl.No	Question	Marks
1	1	Define subbasis of topological space with an example.	2m
1	2	Let A and B denote subsets of a space X. Prove that $\overline{A \cup B} = \overline{A} \cup \overline{B}$.	2m
1	3	Find the boundary and the interior of the subsets $A = \{x \times y / y = 0\}$.	2m
1	4	Show that if $X = \{a, b, c\}$ the collection of all one point subsets of X is a basis for the discrete topology on X.	2m
1	5	Prove that one point sets are closed in Haussdorff space.	2m
1	6	If $\mathcal{B}$ is a basis for the topology of X and $Y \subset X$. Show that the collection $\mathcal{B}_Y = \{B \cap Y / B \in \mathcal{B}\}$ is a basis for the subspace topology on Y.	2m
1	7	If A is a subset of a topological space. Prove that $\overline{A} = IntA \cup BdA$.	2m
1	8	If A is a subsets of a space X. Prove taht $IntA$ and BdA are disjoint.	2m
1	9	Find the set of all limit points of $A = \left\{\frac{1}{n} : n \in \mathbf{Z}^+\right\}$.	2m
1	10	Define a T_1 space. Is T_1 space Haussdorff? Justify.	2m
1	11	Define Homeomorphism with an example.	2m

2	12	Prove that the composite map of two continuous maps is continuous.	2m
2	13	Suppose that $f : X \to Y$ is continuous. $A \subset X$, and $x \in X$. If x is a limit point of A, is it necessarily true that $f(x)$ a limit point of $f(A)$? Justify.	2m
2	14	If A is a connected subspace of a space X and $A \subset B \subset \overline{A}$. Prove that B is also conneted.	2m
2	15	Show that the unit ball B^n is path connected in $\mathbf{R}^n$.	2m
2	16	Let $Y = [0, 1) \cup (1, 2]$ be a subspace of $\mathbf{R}$ with standard topology. Check wheather Y is connnected or not.	2m
2	17	Define a locally path connected space.	2m
2	18	Show that $X = \{0\} \cup \left\{ \frac{1}{n} : n \in \mathbf{Z}^+ \right\}$ of $\mathbf{R}$ is compact.	2m
2	19	Show that every closed subsets of a compact space is compact.	2m
2	20	Show that a component in a topological space is connected.	2m
2	21	Define a quotient map with an example.	2m
3	22	Define a linear Continuom.	2m
3	23	Define a path connected space.	2m
3	24	Define a locally path connected space.	2m
3	25	Is the space $\mathbf{R}_l$ is connected? Why?	2m
3	26	Is compact subset of a topological space is closed? Justify.	2m

4	27	Is the set $(0,1)$ compact in $\mathbf{R}$? Why?	2m
4	28	Show that every closed subset of a compact space is compact.	2m
4	29	Show that $(0,1]$ is not compact in the space $\mathbf{R}$.	2m
4	30	Show that a bijective continuous mapping from a compact space to a Hausdorff space is a homomorphism.	2m
4	31	Give an example of a topological space that is not compact but is locall compact.	2m
4	32	Define Lebesque number.	2m
4	33	Justify with an example that a limit point compact space need not be compact.	2m
1	34	Let X be a set and τ_f be the collection of all subsets U of X such that $X - U$ is either finite or all of X. Prove that τ_f is a topology on X.	5m
1	35	Let X be a topological space. let A be a subset of X. Then prove that $x \in \bar{A}$ if and only if every open set U containg x intersects A.	5m
3	36	Prove that a space X is connected if and only if its only subsets that are both open and closed are $\emptyset$ and X itself.	5m
3	37	Prove that if a collection of connected subsets have a point in common, then the union of elements of the collection is connected.	5m
3	38	Prove that continuous image of a connected space is connected.	5m

3	39	Let X be a topological space. Then prove that each path component of X is contained in a component of X. Also, show that if X is locally path connected, then they are equal.	5m
4	40	State and prove Tube lemma.	5m
4	41	Let X be a compact space and Y be an order set in the order topology. If $f : X \to Y$ is continuous, then prove that there exist $c, d \in X$ such that $f(c) \leq f(x) \leq f(d) \ \ \forall \ \ x \ \in X$.	5m
1	42	Let X be a topological space and Y be a subspace of X. Prove that a subset A of Y is closed in Y if and only if $A = C \cap Y$ where C is some closed set in X.	7m
1	43	Let X be a T_1- space, A be a subset of X and $x \in X$. Then prove that $x \in A'$ if and only if every neighbourhood of x contains infinitely many points of A.	7m
2	44	Let $f : X \to Y$ be a mappig from a metrizable space X into a topological spave Y. Prove that f is continuous if and only if for every sequence $\{x_n\}$ in X converging to x, the sequence $\{f(x_n)\}$ converges to $f(x)$ in Y.	7m
3	45	Let L be a linear continum in the order topology. Then prove that every iterval or a ray in L is connected.	7m

3	46	Let X be a topological space. Prove that the path components of X are disjoint, path connected subsets whose union equals X and each path connected set intersects only one of them.	7m
3	47	Prove that a space is locally connected if and only if has a basis consisting of connected sets	7m
4	48	Prove that the product of finitely many compact space is compact.	7m
4	49	Let Y be a subspace of a topological space X. Prove that Y is compact in subspace topology if and only if every covering of Y by sets open in X has a finite sub-collection that covers Y.	7m
4	50	Let X be a simply ordered set with the least upper bound property. Then prove that every closed interval in X is compact.	7m
4	51	Prove that a subset of $\mathbf{R}^n$ is compact if and only if it is closed and bounded.	7m
4	52	Let X be a Hausdorff space. then prove that X is locally compact if and only if for each $x \in X$, each open set U of x, there is an open set V containing x such that $\overline{V}$ is compact and $\overline{V} \subset U$.	7m
1	53	Show that for any set X, the collection τ generated by the basis $\mathcal{B}$ forms a topology on X. Prove that the lower limit topology on $\mathbb{R}$ is strictly finer than the standard topology on $\mathbb{R}$.	10m

1	54	Let $\mathcal{B}$ and $\mathcal{B}'$ be bases for topologies τ and τ' respectively. Prove that τ' is finer than τ if and only if for each $x \in X$ and each element $B' \in \mathcal{B}'$ such that $x \in B' \subset B$. Let $\mathcal{S}$ be a subbasis for a topology on X. Then prove that the collection $\mathcal{B}$ of all finite intersection of elements of $\mathcal{S}$ form a basis for a topology on X.	10m
1	55	Define the order topology on X. Let X be an ordered set in the ordered topology and Y be a convex subset of X. Then prove that the subspace topology and the order topology on Y are the same. Let X and Y be two topological spaces and $\mathcal{S} = \left\{\pi_1^{-1}(U) : U \textit{ is open in } X\right\} \cup \left\{\pi_2^{-1}(V) : V \textit{ is open in } Y\right\}$. Then prove that $\mathcal{S}$ is a subasis for the product topology on $X \times Y$.	10m
1	56	Let X be a topological space and A be a subset of X. Then prove that $x \in \bar{A}$ if and only if every open set U containing x intersects A. Let X be a topological space and Y be a subspace of X. Prove that a subset A of Y is closed in Y if and only if $A = C \cap Y$ where C is some closed set in X.	10m

2	57	Let X, Y be two topological spaces and $f : X \to Y$ be a map. Then prove that the following are equivalent. (i). f is continuous. (ii). For every subset A of X, $f(\bar{A}) \subset \overline{f(A)}$. (iii). For every closed subset B of Y, $f^{-1}(B)$ is closed in X. State and prove Pasting Lemma.	10m
1	58	Show that X is a Hausdorff space if and only if the diagonal $\triangle = \{x \times x / x \in X\}$ is closed in $X \times X$. Let X be a topological space, $A \subset X$. Then prove that $\overline{A} = A \cup A'$.	10m
2	59	Prove that $\mathbb{R}^n$ is metrizable. Show that composite maps of continuous functions are continuous.	10m
2	60	Let $f : A \to X \times Y$ be given by the equation $f(a) = (f_1(a), f_2(a))$, where $f_1 : A \to X$ and $f_2 : A \to Y$. Prove that f is continuous if and only if f_1 and f_2 are continuous. Prove that arbitraty product of Hausdorff spaces is Hasudorff.	10m
2	61	State and prove the Metrization theorem for $\mathbb{R}^\omega$ under the product topology.	10m
2	62	State and prove sequence lemma. Let $f_n : X \to Y$ be a sequence of continuous functions from a topological space into a metric space. Show that if $f_n \to f$ uniformly, then f is continuous.	10m

3	63	Prove that the product of connected spaces in the product topoloy is connected.	10m
4	64	Let X be a metrizable space. Prove that the following are equivalent: 1. X is compact. 2. X is limit point compact. 3. X is sequentially compact.	10m

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Blue Print of the Question Paper

St. Philomena's College (Autonomous), Mysore

M. Sc-Mathematics (CBCS)

I/II/III/IV- Semester Examination: 2020-21

Subject:

Time: 3 Hours **Max Marks: 70**

Sl. No			Marks
Section – A (MCQ)			
1	a		1
	b		1
	c		1
	d		1
Section – B			
2	a		2
	b		2
	c		2
Section – C Answer any three from the following			
	3		3x10=30
	4		
	5		
	6		
Section – D Answer any three from the following			
	7		3x10=30
	8		
	9		
	10		

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Model Question Paper

St.Philomena's College (Autonomous), Mysuru
M.Sc Mathematics
Third Semester Examination 2021-22
Subject - Topology-I QP Code: 87322

Time - 3 Hours Maximum Marks-70

Section A

Answer the following questions **4 × 1= 4**

1. (a) The lower limit topology on $\mathbb{R}$ is ____ than the standard topology on $\mathbb{R}$.

   (i) not comparable (ii) strictly coarser

   (iii) strictly finer (iv) coarser

   (b) The closure of the set of rational numbers $\mathbb{Q}$ is

   (i) $\mathbb{Q}$ (ii) $\mathbb{C}$ (iii) $\mathbb{R}$ (iv) None of the above

   (c) $\mathbb{R}^\omega$ is metrizable under

   (i) box topology (ii) product topology

   (iii) both (i) and (ii) (iv) None of the above

   (d) Compact subset of a Hausdorff space is

   (i) connected (ii) compact (iii) open (iv) closed

Section B

Answer the following questions **3 × 2= 6**

2. (a) Find the boundary and the interior of the subsets $A = \{x \times y / y = 0\}$.

   (b) Define a T_1 space. Is T_1 space Haussdorff? Justify.

   (c) If A is a connected subspace of a space X and $A \subset B \subset \overline{A}$. Prove that B is also conneted.

Section C

Answer any ***three*** of the following questions **3 × 10= 30**

3. Define the order topology on X. If X is an ordered set in the ordered topology and Y be a convex subset of X, then prove that the subspace topology and the order topology on Y are the same. Further, if X and Y are two topological spaces and $\mathcal{S} = \left\{\pi_1^{-1}(U) : U \textit{ is open in } X\right\} \cup \left\{\pi_2^{-1}(V) : V \textit{ is open in } Y\right\}$. Then prove that $\mathcal{S}$ is a subasis for the product topology on $X \times Y$.

4. Show that X is a Hausdorff space if and only if the diagonal $\triangle = \{x \times x / x \in X\}$ is closed in $X \times X$. And if X is a topological space with $A \subset X$, prove that $\overline{A} = A \cup A'$.

5. Prove that $\mathbb{R}^n$ is metrizable. Show that composite maps of continuous functions are continuous.

6. If $f : A \to X \times Y$ is given by the equation $f(a) = (f_1(a), f_2(a))$, where $f_1 : A \to X$ and $f_2 : A \to Y$. Prove that f is continuous if and only if f_1 and f_2 are continuous. Also, state and prove Pasting Lemma.

Section D

Answer any ***three*** of the following questions **3 × 10= 30**

7. Prove that a space X is connected if and only if its only subsets that are both open and closed are $\emptyset$ and X itself. Further, show that continuous image of a connected space is connected.

8. If X is a topological space, then prove that each path component of X is contained in a component of X. Prove that a space is locally connected if and only if has a basis consisting of connected sets

9. Prove that a subset of $\mathbb{R}^n$ is compact if and only if it is closed and bounded. State and prove Tube lemma.

10. Let X be a metrizable space. Prove that the following are equivalent:

    (i). X is compact.

    (ii). X is limit point compact.

    (iii). X is sequentially compact.

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