St. Philomena's College (Autonomus), Mysuru
PG Department of Mathematics
Question Bank (Revised Curriculum 2020 onwards)
Second Year - Third Semester (2020 -22 Batch)
Course Title (Paper Title): Theory of Numbers Qp. Code: 87331

Unit	Sl.No	Question	Marks
1	1	If P_n is the n^{th} prime, then prove that $P_n \leq 2^{2^{n-1}}$.	2m
1	2	Prove that the only prime p for which $3p+1$ is a perfect square is $p=5$.	2m
1	3	Prove that the only prime of the form n^3-1 is 7.	2m
1	4	If $p \neq 5$ is an odd prime, then prove that p^2-1 or p^2+1 is divisible by 10.	2m
1	5	If $p \geq 5$ is a prime number, show that p^2+2 is composite number.	2m
1	6	Show that Fermat Number F_n for $n=5$ is composite.	2m
1	7	Show that any Fermat Number F_n can be written as the difference of two squares.	2m
1	8	Prove that Fermat Number F_n is never a perfect square.	2m
1	9	For $n>0$, show that Fermat Number F_n is never a triangular number.	2m
1	10	Find the successor of $\frac{4}{9}$ in Farey series $\mathfrak{F}_{13}$.	2m
1	11	Find the successor of $\frac{2}{3}$ in Farey series $\mathfrak{F}_{10}$.	2m
1	12	Prove that the numbers of elements in Farey series $\mathfrak{F}_{\mathfrak{n}}$ is $1+\sum_{i=1}^{n}\phi(i)$.	2m
1	13	Find the sum of all elements in Farey series $\mathfrak{F}_{\mathfrak{n}}$.	2m
2	14	Show that $\phi(p^\alpha)=p^\alpha-p^{\alpha-1}$ for prime p and $\alpha \geq 1$ where ϕ is Euler toient functions.	2m
2	15	Show that identity function is completely multiplicative.	2m
2	16	Show that Mӧbius function is multiplicative but not completely multiplicative.	2m
2	17	Show that f and g are completely multiplicative, then show that fg is completely multiplicative.	2m

2	18	Show that the Mangolt function is not multiplicative function.	2m
3	19	If $\frac{p_n}{q_n}, \frac{p_{n-1}}{q_{n-1}}$, are the n^{th} and $(n-1)^{th}$ convergence of a continued fraction $[a_0, a_1, a_2, \ldots, a_n]$, then show that $\dfrac{p_n}{q_n} - \dfrac{p_{n-1}}{q_{n-1}} = \dfrac{(-1)^n}{q_n\, q_{n-1}}$.	2m
3	20	If $\frac{p_n}{q_n}, \frac{p_{n-2}}{q_{n-2}}$, are the n^{th} and $(n-2)^{th}$ convergence of a continued fraction $[a_0, a_1, a_2, \ldots, a_n]$, then show that $\dfrac{p_n}{q_n} - \dfrac{p_{n-2}}{q_{n-2}} = \dfrac{(-1)^n a_n}{q_n\, q_{n-2}}$.	2m
3	21	Determine rational number represented by the simple continued fraction $[4; 2, 1, 3, 1, 2, 4]$.	2m
3	22	Determine rational number represented by the simple continued fraction $[0; 1, 2, 3, 4, 3, 2, 1]$.	2m
3	23	Express the rational number as finite simple continued fractions $\frac{187}{57}$.	2m
3	24	Express the rational number as finite simple continued fractions $\frac{71}{55}$.	2m
3	25	Compute the convergence of the simple continued fraction $[1, 2, 3, 3, 2, 1]$.	2m
3	26	Compute the convergence of the simple continued fraction $[-3, 1, 1, 1, 1, 3]$.	2m
3	27	By means of continued fraction determine the general solutions of the Diophantine equation $19x + 51y = 1$.	2m
3	28	By means of continued fraction determine the general solutions of the Diophantine equation $18x + 5y = 24$.	2m
4	29	Evaluate the infinite simple continued fraction $[\overline{2; 3}]$.	2m
4	30	Evaluate the infinite simple continued fraction $[1; \overline{2}]$.	2m
4	31	Determine the infinite continued fraction representaion of irrational number $\sqrt{7}$.	2m
4	32	Determine the infinite continued fraction representaion of irrational number $\dfrac{1+\sqrt{13}}{2}$.	2m
4	33	Is $\sqrt{2}$ equivalent to $\sqrt{3}$? Justify your answer.	2m
4	34	Is $\sqrt{2}$ equivalent to $1 + \sqrt{2}$? Justify your answer.	2m

4	35	Show that the product of two odd primes is never a perfect number.	2m
4	36	Show that every even perfect number is triangular number.	2m
4	37	If n is a perfect number then show that $\sum_{d/n} \frac{1}{d} = 2$.	2m
4	38	Show that no powers of a prime can be perfect.	2m
1	39	State and prove the Fundamental theorem of Arithmetic.	6m
1	40	Show that there are infinitely many primes.	4m
1	41	State prime number theorem. Show that $p_n \sim n \log n$ where p_n denote the n^{th} prime.	6m
1	42	Show that the Fermat number $(F_n, F_m) = 1$ for $n \neq m$.	4m
1	43	State and prove Pepin's Test.	10m
1	44	Show that the series $\sum_{p-prime} \frac{1}{p}$ is divergent.	10m
1	45	Show that $\sqrt[m]{N}$ is irrationals unless N is the m^{th} power of an integer.	4m
1	46	Prove that e^y is irrational for any rational $y \neq 0$.	6m
1	47	Show that π^2 is irrational.	6m
1	48	Show that the number e is irrational.	4m
1	49	If $\frac{h}{k}$ and $\frac{h'}{k'}$ are two successive terms of Farey series $\mathfrak{F}_\mathfrak{n}$, then show that $kh' - hk' = 1$.	6m
1	50	If $n > 1$, show that no two successive terms of Farey series $\mathfrak{F}_\mathfrak{n}$ have the same denominator.	4m
2	51	Prove that $\sum_{d/n} \mu(d) = \begin{cases} 1, & \text{if } n = 1 \\ 0, & \text{if } n > 1, \end{cases}$ where $\mu(n)$ is the Mӧbius function.	4m
2	52	If $\phi(n)$ is Euler's functions, then show that $\sum_{d/n} \phi(d) = n$.	6m
2	53	For $n \geq 1$, show that the Euler function $\phi(n) = n \prod_{p/n} \left(1 - \frac{1}{p}\right)$.	6m

2	54	For any two integers m, n, show that $\phi(m.n) = \phi(m).\phi(n).\dfrac{d}{\phi(d)}$, where $d = \gcd(m, n)$.	4m
2	55	Show that $\phi(n) = \sum_{d/n} \mu(d)\ \dfrac{n}{d}$, where $\phi(n)$ is Euler function and $\mu(n)$ is Mӧbius function .	6m
2	56	Prove that the set of all arithmetic functions f with $f(1) \neq 0$ forms an abelian group with respect to the Dirichlet multiplication.	10m
2	57	State and prove Mӧbius inversion formula.	6m
2	58	Show that $\sum_{d\|n} \Lambda(d) = \log n$ for $n \geq 1$, where $\Lambda(n)$ is Mangoldt function.	6m
2	59	Show that for $n \geq 1$, $\Lambda(n) = \sum_{d\|n} \mu(d) \cdot \log \dfrac{n}{d} = -\sum_{d\|n} \mu(d) \cdot \log d$.	4m
2	60	Given f with $f(1) = 1$. Show that f is multiplicative if and only if $f(p_1^{\alpha_1}, p_2^{\alpha_2}, \ldots, p_k^{\alpha_k}) = f(p_1^{\alpha_1})\ f(p_2^{\alpha_2}) \cdots f(p_k^{\alpha_k})$, for all primes p_i and all integers α_i.	6m
2	61	Given f with $f(1) = 1$. If f is multiplicative . Then show that f is completely multiplicative if and only if $f(p^\alpha) = f(p)^\alpha$.	4m
2	62	If f and g are multiplicative, then show that their Dirichlet product $f * g$ is also multiplicative.	6m
2	63	If both g and $f * g$ are multiplicative. Show that f is also multiplicative.	6m
2	64	If f is multiplicative, then show that f is completely multiplicative if and only if $f^{-1}(n) = \mu(n)\ f(n)\ \ \forall\ n \geq 1$.	6m
3	65	If p_n and q_n are defined by $p_0 = a_0$, $p_1 = a_1$, $p_n = a_n p_{n-1} + p_{n-2}$, for $n \geq 2$, and $q_0 = 1$, $q_1 = a_1$, $q_n = a_n q_{n-1} + q_{n-2}$, for $n \geq 2$. Show that $[a_0, a_1, a_2, \cdots, a_n] = \frac{p_n}{q_n}$.	6m
3	66	If $a_1, a_2, \ldots, a_n$ are positive reals, then show that even convergent $\dfrac{p_{2n}}{q_{2n}}$ increase strictly with n, while odd convergent $\dfrac{p_{2n-1}}{q_{2n-1}}$ decrease strictly with n.	4m
3	67	Show that every odd convergent is greater than any even convergent.	6m

3	68	If two simple continued fractions $[a_0, a_1, a_2, \cdots, a_N]$ and $[b_0, b_1, b_2, \cdots, b_M]$ have the same value and $a_N > 1$, $b_M > 1$, then prove that $N = M$ and the continued fractions are identical.	6m
3	69	Show that the convergent to a simple continued fraction are in their lowest terms.	4m
3	70	Prove that any rational number can be represented by a finite simple continued fraction.	6m
3	71	Find the value of the continued fraction $[-2; 1, 2, 5, 7, 4, 1, 6]$.	4m
3	72	Explain a method to determine the general solution of a linear Diophantine equation $ax + by = c$, where a, b, c are integers.	6m
3	73	Solve the linear Diophantine equation $172x + 50y = 500$.	4m
4	74	Show that any infinite simple continued fraction $[a_0, a_1, a_2, \ldots,]$ converges.	6m
4	75	Show that two infinite simple continued fractions which have the same value are identical.	6m
4	76	Show that every irrational number can be expressed in just one way as an infinite simple continued fraction.	6m
4	77	Show that the value of any infinite simple continued fraction is an irrational number.	4m
4	78	Define Equivalent numbers. Show that the relation is an equivalence relation.	4m
4	79	Show that any two rational numbers are equivalent.	4m
4	80	Show that two irrational numbers ξ and η are equivalent if and only if $\xi = [a_0, a_1, a_2, \ldots a_m, c_0, c_1, c_2, \ldots,]$ and $\xi = [b_0, b_1, b_2, \ldots b_n, c_0, c_1, c_2, \ldots,]$.	6m
4	81	Show that a periodic continued fraction is a quadratic surd.	4m
4	82	Show that the continued fraction which represents a quadratic surd is periodic.	10m

4	83	State and prove Hurwitz theorem.	10m
4	84	Define perfect number. If $2^k - 1$ is prime $(k > 1)$ then prove that $2^{k-1}(2^k - 1)$ is perfect and every even perfect number is of this form.	10m
4	85	Show that an even perfect number ends in the digit 6 or 8.	6m
4	86	Prove that $2^{10}(2^{11} - 1)$ is not a perfect number.	4m
4	87	Show that a perfect square can't be a perfect number.	4m
4	88	For any even perfect number $n > 6$. Show that the sum of the digits of n is congruent to $1 \pmod 9$.	6m
4	89	If $n > 6$ is an even perfect number. Show that n can be expressed as sum of consecutive odd cubes.	6m

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Blue Print of the Question Paper

St. Philomena's College (Autonomous), Mysore

M. Sc-Mathematics (CBCS)

I/II/III/IV- Semester Examination: 2020-21

Subject:

Time: 3 Hours **Max Marks: 70**

Sl. No			Marks
Section – A (MCQ)			
1	a		1
	b		1
	c		1
	d		1
Section – B			
2	a		2
	b		2
	c		2
Section – C **Answer any three from the following**			
	3		3x10=30
	4		
	5		
	6		
Section – D **Answer any three from the following**			
	7		3x10=30
	8		
	9		
	10		

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Model Question Paper

St.Philomena's College (Autonomous), Mysuru
M.Sc Mathematics
Third Semester Examination 2021-22
Subject - Theory of Numbers   Qp Code: 87331

Time - 3 Hours   Maximum Marks-70

Section A

Answer the following questions   **4 × 1= 4**

1. (a) The only prime of the form $n^3 - 1$ is

   (i) 7   (ii) 9   (iii)5   (iv) 11

   (b) Identity function is

   (i) completely multiplicative   (ii) multiplicative

   (iii) Neither completely multiplicative nor multiplicative

   (iv) None of the above

   (c) The convergence of the simple continued fraction $[3, 1, 2]$ is

   (i) $\frac{11}{2}$ (ii) $\frac{11}{3}$ (iii) $\frac{13}{4}$ (iv) $\frac{13}{3}$

   (d) The infinite simple continued fraction $[\overline{1}]$ converges to

   (i) $\frac{1+\sqrt{7}}{2}$ (ii) $\frac{1+\sqrt{5}}{2}$ (iii) $\frac{1+\sqrt{5}}{3}$ (iv) $\frac{1+\sqrt{7}}{3}$

Section B

Answer the following questions   **3 × 2= 6**

2. (a) If $p \neq 5$ is an odd prime, then prove that $p^2 - 1$ or $p^2 + 1$ is divisible by 10.

   (b) Compute the convergence of the simple continued fraction $[-3, 1, 1, 1, 1, 3]$.

   (c) Is $\sqrt{2}$ equivalent to $\sqrt{3}$? Justify your answer.

Section C

Answer any ***three*** of the following questions **3 × 10= 30**

3. (a) State and prove the Fundamental theorem of Arithmetic. 6

   (b) Show that there are infinitely many primes. 4

4. (a) Show that the series $\sum_{p-prime} \frac{1}{p}$ is divergent. 10

5. (a) For $n \geq 1$, show that the Euler function $\phi(n) = n \prod_{p/n} \left(1 - \frac{1}{p}\right)$ 6

   (b) For any two integers m, n, show that $\phi(m.n) = \phi(m).\phi(n).\frac{d}{\phi(d)}$, where $d = \gcd(m, n)$. 4

6. (a) Given f with $f(1) = 1$. Show that f is multiplicative if and only if $f(p_1^{\alpha_1}, p_2^{\alpha_2}, \ldots, p_k^{\alpha_k}) = f(p_1^{\alpha_1})\ f(p_2^{\alpha_2}) \cdots f(p_k^{\alpha_k})$, for all primes p_i and all integers α_i. 6

   (b) Given f with $f(1) = 1$. If f is multiplicative . Then show that f is completely multiplicative if and only if $f(p^\alpha) = f(p)^\alpha$. 4

Section D

Answer any ***three*** of the following questions **3 × 10= 30**

7. (a) Show that every odd convergent is greater than any even convergent. 6

   (b) Find the value of the continued fraction $[-2; 1, 2, 5, 7, 4, 1, 6]$. 4

8. (a) Prove that any rational number can be represented by a finite simple continued fraction. 6

   (b) Solve the linear Diophantine equation $172x + 50y = 500$. 4

9. (a) Show that two infinite simple continued fractions which have the same value are identical. 6

(b) Show that the value of any infinite simple continued fraction is an irrational number. 4

10. (a) Define perfect number. If $2^k - 1$ is prime $(k > 1)$ then prove that $2^{k-1}(2^k - 1)$ is perfect and every even perfect number is of this form. 10

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