St.Philomena's College (Autonomus), Mysore

PG Department of Mathematics

Question Bank (Revised Curriculum 2020 onwards)

Second Year - Third Semester (2020 - 22 Batch)

Course Title (Paper Title): Graph Theory Q.P.Code-87332

Unit	S.No	Question	Marks
1	1	Define order and size of a graph. Find order and size of complete graph.	2m
1	2	Draw all graphs with four points.	2m
1	3	Define multi-graph and pseudograph with example.	2m
1	4	What is the maximum number of lines in a graph with p points? Justify.	2m
1	5	Draw two different (5,5) graph.	2m
1	6	Draw a graph with point representing the numbers $1, 2, 3, ..., 10$ in which two points are adjacent if and only if they have a common divisor greater than one.	2m
1	7	Define degree of a point in a graph. Draw all graphs on $p = 3$ points.	2m
1	8	Define minimum degree of a graph with example.	2m
1	9	Define Maximum degree of a graph with example.	2m
1	10	Define regular graph. Draw all regular graph with four points.	2m
1	11	Define cubic graph. Draw all cubic graph with eight points.	2m

1	12	Define complete graph with example.	2m
1	13	Define bridge point of a graph with example.	2m
1	14	For any connected graph G prove that $r(G) \leq diam(G) \leq 2r(G)$.	2m
1	15	Define isomorphism of two graphs. Draw all non-isomorphic graphs with three points.	2m
1	16	Define complement and self complement of a graph with example.	2m
1	17	Define cut point of a graph with example.	2m
1	18	Show that a (p,q) graph is a complete graph if and only if $q = \frac{p(p-1)}{2}$.	2m
1	19	If v is a cut point of G then show that v is not a cut point of $\overline{G}$.	2m
1	20	Define spanning subgraph of a graph. Draw all spanning subgraph of K_3.	2m
1	21	Define induced subgraph of a graph. Draw two induced subgraph of K_5.	2m
1	22	Is complement of a complete graph is a subgraph of that graph? Justify.	2m
1	23	Define complete bipartite graph. Give an example of a bipartite graph which is regular.	2m
1	24	Define block and end block of a graph with example.	2m
1	25	Define distance between two points with example.	2m
1	26	Construct a graph with $\kappa(G) = 3$ $\lambda(G) = 4$ and $\delta(G) = 5$.	2m
1	27	Define block index of point in a graph and find the block index of an isolated point and a cut point.	2m

1	28	Define block graph of a graph. Draw block graph of $K_{1,6}$	2m
1	29	Draw two different graphs G_1 and G_2 such that block graph of G_1 is isomorphic to block graph of G_2.	2m
1	30	Define cut point graph of a graph with example.	2m
1	31	Define point connectivity of a graph and find the point connectivity of $K_{m,n}$.	2m
1	32	Define line connectivity of a graph find the line connectivity of cycle with p number of points.	2m
1	33	Define tree. Draw all trees with four points.	2m
1	34	Draw all trees with seven points and $\Delta(T) \geq 4$.	2m
2	35	Define Eulerian graph with example.	2m
2	36	Define hamiltonian. Give an example for a non-hamiltonian graph which contains a hamiltonian path.	2m
2	37	Show that any hamiltonian graph has no cut points.	2m
2	38	Define neighborhood of a point with example.	2m
2	39	Define maximal non-hamiltonian graph with example.	2m
2	40	Define closure of a graph with example.	2m
2	41	If G is a graph with $p \geq 3$ points such that $deg(v) \geq \frac{p}{2}$ for every point in G then prove that G is hamiltonian.	2m

2	42	Define line graph and write the line graph of $K_4 - e$.	2m
2	43	Define subdivision graph with example.	2m
2	44	Is W_5 is a line graph? Justify.	2m
2	45	Is $K_4 - e$ is a line graph? Justify.	2m
2	46	Draw $L(G)$, $L^2(G)$ for W_4.	2m
3	47	Define factorization of a graph.	2m
3	48	Define planar graph with example.	2m
3	49	Define maximal planar graph with example.	2m
3	50	Give an example for a for a graph G such that both G and $\overline{G}$ are planar.	2m
3	51	Define regular polyhedron with example.	2m
3	52	Define crossing number of a graph with example.	2m
3	53	Find the number of points and lines in complete bipartite graph.	2m
3	54	State Euler's polyhedron formula.	2m
3	55	Define the point covering number of a graph with example.	2m
3	56	Define the line covering number of a graph with example.	2m
3	57	Define the point independence number of a graph with example.	2m
3	58	Define chromatic number of a graph with example.	2m
3	59	Find the chromatic number of complete bipartite and wheel graphs.	2m

3	60	If H is a subgraph of G then prove that $\chi(G) \geq \chi(H)$.	2m
4	61	Define adjacency matrix with example.	2m
4	62	Define incidence matrix with example.	2m
4	63	Define point covering number of a graph with example	2m
4	64	Define cycle matrix with example.	2m
4	65	Mention at least two differences between adjacency matrix and incidence matrix.	2m
4	66	Give an example of dominating set D such that D is common dominating set for C_5 and $\overline{C_5}$.	2m
4	67	Give an example of dominating set D such that D is common dominating set for K_5 and $\overline{K_5}$.	2m
4	68	Give an example for a minimal dominating set need not be minimum.	2m
4	69	Define minimal dominating set with example.	2m
4	70	Define domination number of a graph G with example.	2m
4	71	Find domination number of K_p and $\overline{K_p}$.	2m
4	72	Find domination number of $K_{m,n}$ and $\overline{K_{m,n}}$.	2m
4	73	Find domination number of C_n and P_n.	2m
4	74	Define minimal dominating set with example	2m
1	75	State and prove First theorem of graph theory.	4m

2	76	Prove that $K_{1,3}$ is not a line graph.	4m
2	77	The line graph of a graph G is path if and only if G is path.	4m
2	78	If G is a (p,q) graph then show that $L(G)$ is a (q,q_L) graph where, $q_L = \frac{1}{2}\sum_{i=1}^{p} d_i^2 - q$.	4m
2	79	If G is hamiltonian graph then prove that for every non empty proper subset S of V(G), $w(G-S) \leq \|S\|$, where w(G-S) denotes the number of components in any graph G-S.	4m
2	80	Prove that a graph is Hamiltonian if and only if its closure is Hamiltonian.	4m
3	81	If G is a planar (p,q) graph with r-regions and k-components, then prove that p-q+r=k+1.	4m
3	82	If G is a maximal planar (p,q) graph with $p \geq 3$ then prove that q=3p-6.	4m
3	83	If G is a planar (p,q) graph with $p \geq 3$ then prove that $q \leq 3p-6$.	4m
3	84	Prove that every planar graph contain a point of degree at most 5.	4m
3	85	Prove that the graph K_5 and $K_{3,3}$ are non-planar.	4m
3	86	If G is a planar (p,q) graph without triangles and $p \geq 3$ then prove that $q \leq 2p-4$.	4m
3	87	Prove that at least one face of every polyhedron is bounded by n-cycle for some n=3,4,5,...	4m

3	88	If G is a(p,q) planar graph in which every region is a n-cycle, then prove that $q = \dfrac{n(p-2)}{n-2}$.	4m
3	89	If G is a (p,q) graph then prove that $\chi(G) \geq \dfrac{p^2}{p^2-2q}$.	4m
4	90	Prove that every non trivial connected graph G has a dominating set D whose component $V-D$ is also a dominating set.	4m
1	91	Prove that in any graph the number of point of odd degree is even.	5m
1	92	Prove that every non trivial tree has at least two points of degree one.	5m
1	93	Prove that every non-trivial connected graph with p points has at most p-2 cut points	5m
1	94	Prove that every tree has a center consisting of either one point or two adjacent points.	5m
1	95	Prove that every connected graph has a spanning tree.	5m
1	96	Prove that a line x of a connected graph G is a bridge if and only if it is not on any cycle of G.	5m
1	97	Prove that a cubic graph has a cut point if and only if it has a bridge.	5m
1	98	For any graph G prove that $b(G) = c(G) + \sum_{v \in V(G)} [b(v)-1]$ where, b(G) is the number of blocks of G and c(G) is the number of components of G.	5m
1	99	Prove that a graph H is the block graph of some graph if and only if every block of H is complete.	5m

1	100	Prove that among all graphs with p points and q lines, the maximum connectivity is zero when $q < p-1$ and is $\left\lfloor \dfrac{2q}{p} \right\rfloor$ when $q \geq p-1$, where $\lfloor x \rfloor$ is the greatest integer less than or equal to x.	5m
1	101	Prove that if G is a k-connected (p,q) graph then $q \geq \dfrac{pk}{2}$. Check whether there is 5-connected graph with 20 lines.	5m
2	102	A graph is the line graph of a tree if and only if it is a connected block graph in which each cut point is on exactly two blocks.	5m
3	103	Prove that the complete graph K_{2n}, $n \geq 1$ is 1-factorization. Display a 1- factorization for K_8.	5m
3	104	Prove that the graph K_{2n+1}, $n \geq 1$ is the sum of n- spanning cycles. Display 4- hamiltonian cycle for K_9.	5m
3	105	Prove that for any graph G, $\chi(G) \leq 1 + max\delta(G')$ where the maximum is taken over all induced subgraphs G' of G.	5m
3	106	Prove that in a connected graph G there is a Eulerian trail if and only if the number of points of odd degree is either zero or two.	5m
3	107	State and prove Euler's formula of a connected planar graph.	5m
3	108	Prove that the graph K_5 and $K_{3,3}$ are non-planar.	5m
3	109	Let G be a connected graph with 2n points, $n \geq 1$ odd points, then prove that the set of lines of G can be partitioned into n open trials.	5m
3	110	If every block of a connected graph G is Eulerian, then G is Eulerian.	5m

3	111	Let G be a maximal planar graph of order $p \geq 4$ and let p_i denote the number of points of degree i, for $i = 3, 4, ..., n = \Delta(G)$ then prove that $3p_3 + 2p_4 + p_5 = p_7 + 2p_8 + \cdots + (n-6)p_n + 12$	5m
3	112	For any graph G prove that $\frac{p}{\beta} \leq \chi(G) \leq p - \beta + 1$. where β is the independence number of G.	5m
4	113	Demonstrate matrix tree theorem on $K_4 - e$.	5m
4	114	A graph G on p points is connected if and only if $(A+I)^{p-1}$ has no zero entries.	5m
4	115	If $A = (a_i j)$ be the adjacency matrix of a graph G then prove that $(i,j)^{th}$ entry in A^n $[(A^n)_{ij}]$ is the number of walks of length n from v_i to v_j with example.	5m
4	116	If G has incidence matrix B and cycle matrix C then $CB^T \equiv 0(mod 2)$ where, B^T is the transpose of B.	5m
4	117	Find the labelled spanning trees on p points using matrix tree theorem.	5m
4	118	Prove that the following graphs are equivalent for any graph G (i) G is 2-colorable. (ii) G is bipartite.	5m
4	119	If G is a graph of order n then prove that $\left\lfloor \frac{n}{1+\Delta(G)} \right\rfloor \leq \gamma(G) \leq n - \beta$.	5m
4	120	If G is a graph having p points and q lines then prove that $p - q \leq \gamma(G) \leq p - \Delta$.	5m

4	121	If G be a graph without isolated points and D is a minimal dominating set then show that $V - D$ is a dominating set.	5m
4	122	If G be any graph then show that $p - q \leq \gamma(G)$, further $\gamma(G) = p - q$ if and only if each component of G is a star.	5m
4	123	Prove that there exists a k-coloring of a graph G if and only if V(G) can be partitioned into k subsets $V_1, V_2, ..., Vk$ such that no two points in V_i, i=1,2,...,k are adjacent.	5m
1	124	Let G be a (p,q) graph then prove that $\delta \leq \frac{2q}{p} \leq \Delta$. Does there exist a 3- regular graph with six points? If so construct the graph.	6m
1	125	Let v be a point of a connected graph G then prove that the following statements are equivalent i) v is a cut point of G. ii) There exist a partition of $V - \{v\}$ into two subsets U and W such that for any point $u \in U$ and $w \in W$ the point v lies on every u-w path. iii) There exist two points u and w distinct from v such that v is on every u-w path.	6m
1	126	Let G be a connected graph with at least three points then prove that the following statements are equivalent i) G is a block. ii) Any two points of G lie on a common cycle. iii) Any point and any line of G lie on a common cycle. iv) Any two lines of G lie on a common cycle.	6m

1	127	Let G be a connected graph with at least three points then prove that the following statements are equivalent i) Every two lines of G lie on a common cycle. ii) Given two points and one line of G there is a path joining the points which contains the line. iii) For every three distinct point of G there is a path joining any two of them which contains the third. iv) For every three distinct point of G there is a path joining any two of them which doesn't contains the third.	6m
1	128	For any graph G prove that $\kappa(G) \leq \lambda(G) \leq \delta(G)$.	6m
1	129	If graph G has p points and $\delta(G) \geq \frac{p}{2}$ then prove that $\lambda(G) = \delta(G)$.	6m
1	130	Let G be a (p,q) graph then prove that the following statements are equivalent i) G is a tree. ii) Every two points of G are joined by unique path. iii) G is connected and p=q+1. iv) G is acyclic and p=q+1.	6m
2	131	Show that the following statements are equivalent i). G is a line graph. ii). The line of G can be partitioned into complete subgraphs in such a way that no points lies in more than two of the subgraphs.	6m
2	132	Prove that the following statements are equivalent for a connected graph G, (i) G is Eulerian. (ii) Every point of G has even degree. (iii) The set of lines of G can be partitioned into cycles.	6m
3	133	For any graph G, the sum and product of χ and $\overline{\chi}$ satisfy the inequalities $2\sqrt{p} \leq \chi + \overline{\chi} \leq p + 1$ and $p \leq \chi\overline{\chi} \leq \frac{(p+1)^2}{4}$.	6m

3	134	If G is a graph with $p \geq 3$ points and $\delta(G) \geq \frac{p}{2}$ then prove that G is hamiltonian.	6m
3	135	Let G be a graph with p points and let u and v be non-adjacent points in G such that $deg(u) + deg(v) \geq p$. Then prove that G is hamiltonian if and only if G+uv is hamiltonian.	6m
3	136	Prove that there are five regular polyhedron.	6m
4	137	For any graph with incidence matrix B, show that $A(L(G)) = B^T B - 2I_q$, where B^T is a transpose of B.	6m
4	138	A dominating set D is a minimal dominating set if and only if for each vertex v in$D$, one of the following condition holds i) v is an isolated vertex of D. ii)there exist a vertex u in $V - D$ such that $N(x) \cap D = \{v\}$.	6m
4	139	prove that for any (p, q) graph without isolated point $\gamma(G) \leq \alpha(G)$. where $\alpha(G)$ is point covering number of G.	6m

St.Philomena's College (Autonomus), Mysore

M. Sc. - Mathematics (CBCS)

Model Question Paper

III- Semester Examination: 2020-21

Subject: Graph Theory

Time: 3 Hours Max Marks : 70

Section - A (MCQ)

1. (a) The cycle on n vertices is isomorphic to its compliment. The value of n is

   i) 5 ii) 4

   iii) 6 iv) 3 1

   (b) In a connected graph, a bridge is an edge whose removal disconnects a graph. Which one of the following statement is true.

   i) A tree has no bridges

   ii) A bridge can not be part of a simple cycle

   iii) A graph with bridges cannot have a cycle

   iv) none of the above 1

   (c) Let G be a connected planar graph with 10 vertices. If the number of edges on each face is 3, then the number of edges in G is

i) 20 ii) 24

iii) 16 iv) 26 1

(d) Let A be a adjacency matrix of a simple graph. Which one of the following statement is not related to A.

i) Every diagonal entries are zero

ii) Symmetric matrix

iii) i^{th} row as well as i^{th} column of the matrix is equal to the degree of v_i

iv) Every column of the matrix contains exactly two 1's 1

Section - B

2. (a) Show that a $G = (p, q)$ graph is a complete graph if and only if $q = \frac{p(p-1)}{2}$. 2

(b) Is $K_4 - e$ is a line graph? Justify. 2

(c) Define domination number of a graph G with example. 2

Section - C

Answer any three from the following. $3 \times 10 = 30$

3. (a) Prove that in any graph the number of point of odd degree is even. 5

(b) Prove that every connected graph has a spanning tree. 5

4. (a) Prove that a graph H is the block graph of some graph if and only if every block of H is complete. 5

   (b) Prove that if G is a k-connected (p,q) graph then $q \geq \frac{pk}{2}$. Check whether there is 5-connected graph with 20 lines. 5

5. (a) Prove that $K_{1,3}$ is not a line graph. 4

   (b) Show that the following statements are equivalent

      i. G is a line graph.

      ii. The line of G can be partitioned into complete subgraphs in such a way that no points lies in more than two of the subgraphs. 6

6. (a) Prove that the following statements are equivalent for a connected graph G,

      i. G is Eulerian.

      ii. Every point of G has even degree.

      iii. The set of lines of G can be partitioned into cycles. 6

   (b) If G is hamiltonian graph then prove that for every non empty proper subset S of V(G), $w(G-S) \leq |S|$, where w(G-S) denotes the number of components in any graph G-S. 4

Section - D

Answer any three from the following. $3 \times 10 = 30$

7. (a) Prove that the graph K_5 and $K_{3,3}$ are non-planar. 4

   (b) For any graph G, the sum and product of χ and $\overline{\chi}$ satisfy the inequalities $2\sqrt{p} \leq \chi + \overline{\chi} \leq p+1$ and $p \leq \chi\overline{\chi} \leq \dfrac{(p+1)^2}{4}$. 6

8. (a) If G is a (p,q) graph then prove that $\chi(G) \geq \dfrac{p^2}{p^2 - 2q}$. 4

   (b) Prove that there are five regular polyhedron. 6

9. (a) Prove that every non trivial connected graph G has a dominating set D whose component $V-D$ is also a dominating set. 4

   (b) For any graph with incidence matrix B, show that $A(L(G)) = B^T B - 2I_q$, where B^T is a transpose of B. 6

10. (a) Demonstrate matrix tree theorem on $K_4 - e$. 4

    (b) A dominating set D is a minimal dominating set if and only if for each vertex v in D, one of the following condition holds

    i. v is an isolated vertex of D.

    ii. there exist a vertex u in $V-D$ such that $N(x) \cap D = \{v\}$. 6